حل - تشریحی

-۱

$y = \tan -۲\left(x - \frac{\pi}{۸}\right)$

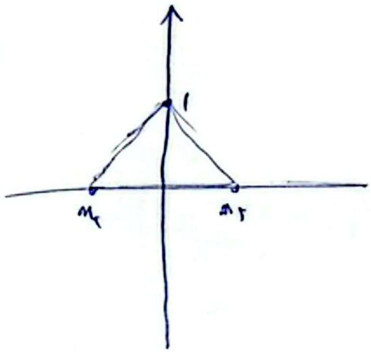
$\tan\left(۲k\pi + \frac{\pi}{۲}\right) = \tan -۲\left(x - \frac{\pi}{۸}\right)$ $\tan\left(۲k\pi + \frac{۳\pi}{۲}\right) = \tan -۲\left(x - \frac{\pi}{۸}\right)$

$-۲\left(x - \frac{\pi}{۸}\right) = ۲k\pi + \frac{\pi}{۲}$ $x_1 = -k\pi - \frac{\pi}{۴} + \frac{\pi}{۸}$ $-۲\left(x - \frac{\pi}{۸}\right) = ۲k\pi + \frac{۳\pi}{۲}$

$x_۲ = -k\pi - \frac{۳\pi}{۴} + \frac{\pi}{۸}$

$\tan -۲\left((۰) - \frac{\pi}{۸}\right) = \tan\left(\frac{\pi}{۴}\right) = 1$

$x_۲ - x_1 = \cancel{-k\pi} - \frac{۳\pi}{۴} + \cancel{\frac{\pi}{۸}} + \cancel{k\pi} + \frac{\pi}{۴} - \cancel{\frac{\pi}{۸}} = \frac{\pi}{۲}$



$\boxed{S_\Delta = \frac{1}{۲} \times 1 \times \frac{\pi}{۲} = \frac{\pi}{۴}}$

۲

$x^۲ - (\sin\alpha)x - \frac{1}{۴}\cos^۲\alpha = ۰$ $\frac{۴}{۹} - \frac{۲}{۳}\sin\alpha - \frac{1}{۴}\left(\cos^۲\alpha\right) = ۰ \rightarrow \frac{۴}{۹} - \frac{۲}{۳}\sin\alpha - \frac{1}{۴}\left(1 - \sin^۲\alpha\right) = ۰$

$\cancel{-\frac{۲}{۳}\sin\alpha - \frac{1}{۴}\left(\frac{1}{1+\tan^۲\alpha}\right) = -\frac{۴}{۹}}$ $\frac{1}{۴}\sin^۲\alpha - \frac{۲}{۳}\sin\alpha + \frac{۷}{۳۶} = ۰$

$\cancel{\frac{1}{۴}\left(\frac{1}{1+\tan^۲\alpha}\right) = -\frac{۴}{۹} - \frac{۴}{۹}\sin\alpha}$

$\frac{1}{۴}\sin^۲\alpha - \frac{۲}{۳}\sin\alpha + \frac{۷}{۳۶} = ۰ \;\times ۳۶$ $۹\sin^۲\alpha - ۲۴\sin\alpha + ۷ = ۰$ $\left(\sin\alpha - \frac{۲۱}{۹}\right)\left(\sin\alpha - \frac{۳}{۹}\right) = ۰$

$\sin\alpha = \frac{۷}{۳}$ غ ق ق $\boxed{\sin\alpha = \frac{1}{۳}\ \text{ق ق}}$

$\sin\alpha = \frac{1}{۴}$ $\cos^۲\alpha = \left(1 - \frac{1}{۹}\right) = \frac{۸}{۹} \times \frac{1}{۴}$

$x^۲ - \frac{x}{۳} - \frac{۲}{۹} = ۰$ $\rightarrow$ $\boxed{1 + \tan^۲\alpha = 1 + \frac{\sin^۲\alpha}{\cos^۲\alpha} = 1 + \frac{\frac{1}{۹}}{\frac{۸}{۹}} = 1 + \frac{1}{۸} = \frac{۹}{۸}}$

$\log(-\sin\alpha) - \log(-\cos\alpha) = \log ۳ \qquad \cancel{\log}\, \frac{\sin\alpha}{-\cos\alpha} = \cancel{\log}\, ۳ \quad \frac{\sin\alpha}{-\cos\alpha} = ۳ \quad \tan\alpha = -۳$

$\left.\begin{array}{l} \sin\alpha > ۰ \\ -\cos\alpha > ۰ \quad \cos\alpha < ۰ \end{array}\right\} \Longrightarrow$ ناحیه ۲

$\sin\alpha = \frac{۱}{\sqrt{۱+\cot^۲\alpha}} = \frac{۱}{\sqrt{۱+\frac{۱}{۹}}} = \frac{۱}{\sqrt{\frac{۱۰}{۹}}} = \frac{۳}{\sqrt{۱۰}}$

$\cos\alpha = \frac{۱}{\sqrt{۱+\tan^۲\alpha}} = \frac{۱}{\sqrt{۱۰}} \Rightarrow$

$\sin\alpha - \cos\alpha = \frac{۳}{\sqrt{۱۰}} - \left(-\frac{۱}{\sqrt{۱۰}}\right) = \frac{۴}{\sqrt{۱۰}} = \boxed{\frac{۲\sqrt{۱۰}}{۵}}$

-۴

$\sin^4\theta + \cos^2\theta = \frac{4}{5}$ $\quad \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$ $\quad \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{4}{5}$ $\quad \left(\sin^2\theta - \frac{1}{2}\right)^2 = \frac{1}{20}$ $\quad \sin^2\theta = \frac{10+\sqrt{20}}{20}$

$\sin^2\theta = \frac{10+2\sqrt{5}}{20} = \frac{5+\sqrt{5}}{10}$

$\cos^4\theta + \sin^2\theta = ?$ $\quad (1-\sin^2\theta)^2 + \sin^2\theta = \left(\frac{10-(5+\sqrt{5})}{10}\right)^2 + \left(\frac{5+\sqrt{5}}{10}\right)^2 = \frac{30-10\sqrt{5}}{100} + \frac{30+10\sqrt{5}}{100} = \frac{60}{100} = 0/6$

-۵

$\frac{3\sin x + \sin x\cos x}{2-2\cos^2 x} < 0$ $\quad \frac{\sin x(3+\cos x)}{2(1-\cos^2 x)} < 0$ $\quad \frac{\sin x(3+\cos x)}{2(\sin^2 x)} < 0$

$\sin^2 x > 0$

$3+\cos x > 0$

$\Rightarrow \sin x < 0$

$\sin x \tan x - \frac{1}{\cos x} > 0$ $\quad \frac{\sin^2 x - 1}{\cos x} > 0$ $\quad \frac{1-\cos^2 x - 1}{\cos x} > 0$ $\quad \frac{-\cos^{\cancel{2}} x}{\cancel{\cos x}} > 0$ $\quad -\cos x > 0$

انتهای کمان مثلثاتی در ناحیه سوم است $\quad \cos x < 0$

-۶

$\frac{2}{\sin x} + \frac{3}{\cos x} = 0$ $\quad \frac{2}{\tan x} + 3 = 0$ $\quad \frac{2}{\tan x} = -3$ $\quad \tan x = -\frac{2}{3}$ $\quad \cot x = \frac{1}{\tan x} = -\frac{3}{2}$

$\tan x + \cot x = -\frac{2}{3} + -\frac{3}{2} = \frac{-4+-9}{6} = \boxed{\frac{-13}{6}}$

-۷

$\frac{\sqrt{1+\sin 50^\circ}}{\sin 130^\circ + \sin 10^\circ} = \frac{\sqrt{1+\cos 40^\circ}}{\cos 40^\circ + \cos 80^\circ} = \frac{\sqrt{1+\cos 40^\circ}}{2\cos^2 40^\circ + \cos 40^\circ - 1} = \frac{\sqrt{1+\cos 40^\circ}}{(\cos 40^\circ + 1)\left(\cos 40^\circ - \frac{1}{2}\right)}$

$\cos 80^\circ = 2\cos^2 40^\circ - 1$

$\frac{\cancel{\sqrt{1+\cos 40^\circ}}}{\cancel{(\cos 40^\circ + 1)}\left(\cos 40^\circ - \frac{1}{2}\right)} = \frac{1}{\left(\sqrt{\cos 40^\circ + 1}\right)\left(\cos 40^\circ - \frac{1}{2}\right)} = \frac{1}{\cos 40^\circ + 1} \times \frac{1}{\left(\cos 40^\circ - \frac{1}{2}\right)^2} =$

-۸

$$\left(۱-۲\sin^۲ ۵^\circ\right)\left(۲\cos^۲ ۱۰^\circ-۱\right)\left(\frac{۱-\tan^۲ ۲۰^\circ}{۱+\tan^۲ ۲۰^\circ}\right)$$

$$\frac{\left(\sin^۲ ۵^\circ+\cos^۲ ۵^\circ-۲\sin^۲ ۵^\circ\right)}{\left(\cos ۱۰^\circ\right)}\quad \left(\frac{۲\cos^۲ ۱۰^\circ-\sin^۲ ۱۰^\circ-\cos^۲ ۱۰^\circ}{\left(\cos ۲۰^\circ\right)}\right)\quad\left(\frac{۱-\tan^۲ ۲۰^\circ}{\frac{۱}{\cos^۲ ۲۰^\circ}}\right)$$

$$\left(\cos ۱۰^\circ\right)\left(\cos ۲۰^\circ\right)\left(\frac{\cos^۲ ۲۰^\circ-\sin^۲ ۲۰^\circ}{\frac{۱}{\cos ۴۰^\circ}}\right)=\cos ۱۰^\circ \cos ۲۰^\circ \cos ۴۰^\circ \longrightarrow \times\frac{\sin ۱۰^\circ}{\sin ۱۰^\circ}$$

$$\frac{\overbrace{\sin ۱۰^\circ \cos ۱۰^\circ}^{\frac{۱}{۲}\sin ۲۰^\circ} \cos ۲۰^\circ \cos ۴۰^\circ}{\sin ۱۰^\circ}=\frac{\frac{۱}{۲}\left(\frac{۱}{۲}\sin ۴۰^\circ\right)\cos ۴۰^\circ}{\sin ۱۰^\circ}=\frac{\frac{۱}{۴}\left(\frac{۱}{۲}\sin ۸۰^\circ\right)}{\sin ۱۰^\circ}=\frac{\frac{۱}{۸}\sin ۸۰^\circ}{\sin ۱۰^\circ}$$

$\sin ۸۰^\circ=\cos ۱۰^\circ$

$$\boxed{\frac{۱}{۸}\cot ۱۰^\circ}$$

-۹

$\sin\alpha+۳\cos\alpha=۲$ $\quad$ $\sin\alpha=۲-۳\cos\alpha$

$۵\cos ۲\alpha-۱۲\cos\alpha=۵\left(۲\cos^۲\alpha-۱\right)-۱۲\cos\alpha=۱۰\cos^۲\alpha-۱۲\cos\alpha-۵$

$\sin^۲+\cos^۲=۱$ $\quad$ $\left(۲-۳\cos\alpha\right)^۲+\cos^۲\alpha=۱$

$۴-۱۲\cos\alpha+۹\cos^۲\alpha+\cos^۲\alpha=۱$

$۱۰\cos^۲\alpha-۱۲\cos\alpha+۳=۰$

$\rightarrow ۱۰\cos^۲\alpha-۱۲\cos\alpha+۳-۸=-۸$

$$\boxed{۱۰\cos^۲\alpha-۱۲\cos\alpha-۵=-۸}$$

$\sin\hat{B}\sin\hat{C}=\cos^۲\frac{\hat{A}}{۲}$ $\quad$ $\sin\hat{B}\sin\hat{C}=\frac{۱+\cos\hat{A}}{۲}$ $\quad$ $۲\sin\hat{B}\sin\hat{C}=۱+\cos\hat{A}$

$\hat{B}=۷۲^\circ$

$\cos\hat{A}=\cos\left(\pi-(B+C)\right)=-\cos(B+C)=-\left(\cos B\cos C-\sin B\sin C\right)$

$\cos B\cos C+\sin B\sin C=۱\Longrightarrow\cos(B-C)=۱$

$B-C=۰$

$B=C=۷۲^\circ$ $\quad$ $\boxed{A=۱۸۰-\overbrace{(B+C)}^{۱۴۴}=۳۶^\circ}$