محسن شهبازی

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$y = \tan -2\left(x-\frac{\pi}{8}\right)$

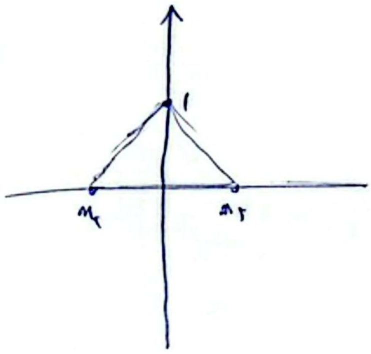
$\tan\left(2k\pi+\frac{\pi}{2}\right) = \tan -2\left(x-\frac{\pi}{8}\right)$ $\qquad$ $\tan\left(2k\pi+\frac{3\pi}{2}\right) = \tan -2\left(x-\frac{\pi}{8}\right)$

$-2\left(x-\frac{\pi}{8}\right) = 2k\pi+\frac{\pi}{2}$ $\quad x_1 = -k\pi-\frac{\pi}{4}+\frac{\pi}{8}$ $\qquad$ $-2\left(x-\frac{\pi}{8}\right) = 2k\pi+\frac{3\pi}{2}$

$x_2 = -k\pi-\frac{3\pi}{4}+\frac{\pi}{8}$

$\tan -2\left((0)-\frac{\pi}{8}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

$x_2-x_1 = -k\pi-\frac{3\pi}{4}+\frac{\pi}{8}+k\pi+\frac{\pi}{4}-\frac{\pi}{8} = \frac{\pi}{2}$



$\boxed{S_\Delta = \frac{1}{2}\times 1\times\frac{\pi}{2} = \frac{\pi}{4}}$

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$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha = 0$ $\qquad$ $\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(\cos^2\alpha) = 0 \to \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha) = 0$

$\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36} = 0$

$\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36} = 0 \;\times 36$ $\quad 9\sin^2\alpha-24\sin\alpha+7 = 0$ $\qquad \left(\sin\alpha-\frac{21}{9}\right)\left(\sin\alpha-\frac{3}{9}\right) = 0$

$\sin\alpha = \frac{7}{3}$ غ ق ق $\qquad \boxed{\sin\alpha = \frac{1}{3} \text{ ق ق}}$

$\sin\alpha = \frac{1}{3}$ $\qquad \cos^2\alpha = \left(1-\frac{1}{9}\right) = \frac{8}{9}\times\frac{1}{4}$

$x^2-\frac{x}{3}-\frac{2}{9} = 0$ $\quad \to \quad \boxed{1+\tan^2\alpha = 1+\frac{\sin^2\alpha}{\cos^2\alpha} = 1+\frac{\frac{1}{9}}{\frac{8}{9}} = 1+\frac{1}{8} = \frac{9}{8}}$

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$\log(-\sin\alpha) - \log(-\cos\alpha) = \log 3 \quad \cancel{\log}\,\frac{\sin\alpha}{-\cos\alpha} = \cancel{\log}\,3 \quad \frac{\sin\alpha}{-\cos\alpha} = 3 \quad \tan\alpha = -3$

$$\left.\begin{array}{l} \sin\alpha > 0 \\ -\cos\alpha > 0 \quad \cos\alpha < 0 \end{array}\right\} \Longrightarrow$$ ناحیه ۲

$$\sin\alpha = \frac{1}{\sqrt{1+\cot^2\alpha}} = \frac{1}{\sqrt{1+\frac{1}{9}}} = \frac{1}{\sqrt{\frac{10}{9}}} = \frac{3}{\sqrt{10}}$$

$$\cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}} = \frac{1}{\sqrt{10}} \Rightarrow$$

$$\sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} - \left(-\frac{1}{\sqrt{10}}\right) = \frac{4}{\sqrt{10}} = \boxed{\frac{2\sqrt{10}}{5}}$$

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$\sin^2\theta^2 + \cos^2\theta = \frac{4}{5}$

$\sin^4\theta + \cos^2\theta = \frac{4}{5}$ $\quad \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$ $\quad (\sin^2\theta - \frac{1}{2})^2 + \frac{3}{4} = \frac{4}{5}$ $\quad (\sin^2\theta - \frac{1}{2})^2 = \frac{1}{20}$ $\quad \sin^2\theta = \frac{10+\sqrt{20}}{20}$

$(1-\cos^2\theta)^2 + \cos^2\theta = \frac{4}{5}$

$\sin^2\theta = \frac{10+2\sqrt{5}}{20} = \frac{5+\sqrt{5}}{10}$

$\cos^4\theta - \cos^2\theta = -\frac{1}{5}$

$\cos^4\theta + \sin^4\theta = ?$ $\quad (1-\sin^2\theta)^2 + \sin^4\theta =$ $\quad \left(\frac{10-(5+\sqrt{5})}{10}\right)^2 + \cancel{\left(\frac{5+\sqrt{5}}{10}\right)^2} = \frac{30-10\sqrt{5}}{100} + \frac{30+10\sqrt{5}}{100} = \cancel{\frac{60}{100}} = 0.6$

$\cos^4\theta - (1-\sin^2\theta) = -\frac{1}{5}$

$\cos^4\theta + \sin^2\theta = \frac{4}{5}$

(1)

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$\frac{3\sin x + \sin x\cos x}{2-2\cos^2 x} < 0$ $\quad \frac{\sin x(3+\cos x)}{2(1-\cos^2 x)} < 0$ $\quad \frac{\sin x(3+\cos x)}{2(\sin^2 x)} < 0$

$\left.\begin{array}{l} \sin^2 x > 0 \\ 3+\cos x > 0 \end{array}\right\} \Rightarrow \sin x < 0$

(2)

$\sin x\tan x - \frac{1}{\cos x} > 0$ $\quad \frac{\sin^2 x - 1}{\cos x} > 0$ $\quad \frac{1-\cos^2 x - 1}{\cos x} > 0$ $\quad \frac{-\cos^2 x}{\cancel{\cos x}} > 0$ $\quad -\cos x > 0$

انتهای کمان مثلثاتی در ناحیه سوم است

$\cos x < 0$

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$\frac{2}{\sin x} + \frac{3}{\cos x} = 0$ $\quad \frac{2}{\tan x} + 3 = 0$ $\quad \frac{2}{\tan x} = -3$ $\quad \tan x = -\frac{2}{3}$ $\quad \cot x = \frac{1}{\tan x} = -\frac{3}{2}$

$\tan x + \cot x = -\frac{2}{3} + -\frac{3}{2} = \frac{-4+-9}{6} = \boxed{\frac{-13}{6}}$

(3)

۷-

$\frac{\sqrt{1+\sin 50^\circ}}{\sin 10^\circ + \sin 10^\circ} = \frac{\sqrt{1+\cos 40^\circ}}{\cos 40^\circ + \cos 80^\circ} = \frac{\sqrt{1+\cos 40^\circ}}{2\cos^2 40^\circ + \cancel{\cos 40^\circ - 1}} = \frac{\sqrt{1+\cos 40^\circ}}{(\cos 40^\circ + 1)(\cos 40^\circ - \frac{1}{2})}$

$\cos 80^\circ = 2\cos^2 40^\circ - 1$

$\sqrt{2\cos^2 20}$

$\frac{\cancel{\sqrt{1+\cos 40^\circ}}}{(\cos 40^\circ + 1)(\cos 40^\circ - \frac{1}{2})} = \frac{1}{(\sqrt{\cos 40^\circ + 1})\cancel{(\cos 40^\circ - \frac{1}{2})}} = \frac{1}{\cos 40^\circ + 1} \times \frac{1}{(\cos 40^\circ - \frac{1}{2})^2} =$

$\sin(20+30)$ $\quad \sin(30-20)$

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$\frac{\sqrt{2}\cos 20}{2\sin 30^\circ \cos 20} = \sqrt{2}$

-8

$$\left(1-2\sin^2 5^\circ\right)\left(2\cos^2 10^\circ-1\right)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)$$

$$\frac{\left(\cancel{\sin^2 5^\circ+\cos^2 5^\circ-2\sin^2 5^\circ}\right)}{\left(\cos 10^\circ\right)}\ \frac{\left(\cancel{2\cos^2 10^\circ-\sin^2 10^\circ-\cos^2 10^\circ}\right)}{\left(\cos 20^\circ\right)}\ \left(\frac{1-\tan^2 20^\circ}{\frac{1}{\cos^2 20^\circ}}\right)$$

$$\left(\cos 10^\circ\right)\left(\cos 20^\circ\right)\left(\frac{\cancel{\cos^2 20^\circ-\sin^2 20^\circ}}{\cancel{1}}\right)_{\cos 40^\circ} = \cos 10^\circ \cos 20^\circ \cos 40^\circ \longrightarrow \times\frac{\sin 10^\circ}{\sin 10^\circ}$$

$$\frac{\overset{\frac{1}{2}\sin 20^\circ}{\cancel{\sin 10^\circ \cos 10^\circ}}\cos 20^\circ \cos 40^\circ}{\sin 10^\circ} = \frac{\frac{1}{2}\left(\frac{1}{2}\sin 40^\circ\right)\cos 40^\circ}{\sin 10^\circ} = \frac{\frac{1}{4}\left(\frac{1}{2}\sin 80^\circ\right)}{\sin 10^\circ} = \frac{\frac{1}{8}\sin 80^\circ}{\sin 10^\circ}$$

$\sin 80^\circ = \cos 10^\circ$

$$\boxed{\frac{1}{8}\cot 10^\circ}$$

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-9

$\sin\alpha + 3\cos\alpha = 2$ $\quad \cancel{\tan\alpha + 3 = \frac{2}{\cos\alpha}}$ $\quad \sin\alpha = 2-3\cos\alpha$

$5\cos 2\alpha - 12\cos\alpha = 5\left(2\cos^2\alpha - 1\right) - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5$

$\sin^2 + \cos^2 = 1 \quad \left(2-3\cos\alpha\right)^2 + \cos^2\alpha = 1$

$4 - 12\cos\alpha + 9\cos^2\alpha + \cos^2\alpha = 1$

$10\cos^2\alpha - 12\cos\alpha + 3 = 0$

$\to 10\cos^2\alpha - 12\cos\alpha + 3 - 8 = -8$

$$\boxed{10\cos^2\alpha - 12\cos\alpha - 5 = -8}$$

2 ✓

$-\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2} \qquad \sin\hat{B}\sin\hat{C} = \frac{1+\cos\hat{A}}{2} \qquad 2\sin\hat{B}\sin\hat{C} = 1+\cos\hat{A}$

$\hat{B} = 72^\circ$

$\cos\hat{A} = \cos\left(\pi-(B+C)\right) = -\cos(B+C) = -\left(\cos B\cos C - \sin B\sin C\right)$

$\cos B\cos C + \sin B\sin C = 1 \Longrightarrow \cos(B-C) = 1$

$B - C = 0$

$B = C = 72^\circ \qquad \boxed{A = 180 - \overset{144}{\cancel{(B+C)}} = 36^\circ}$

2 ✓