محمدعماد عفورانا / دوازدهم ریاضی

20

-1

$T = \frac{\pi}{|-2|} = \pi/2$

$B \Rightarrow x=0 \rightarrow B = \tan \pi/4 = 1$ 2 ✓

$\Rightarrow \frac{AC \times B}{2} \Rightarrow \frac{\pi}{2} \times 1 \times \frac{1}{2} = \pi/4$

-2

$x^2 - \sin\alpha\, x - \frac{1}{4}\cos^2\alpha = 0 \rightarrow 4/9 - 2/3 \sin\alpha - 1/4(1-\sin^2\alpha) = 0$

$4/9 - 2/3 \sin\alpha - 1/4 + 1/4 \sin^2\alpha = 0$

$\Rightarrow 1/4 \sin^2\alpha - 2/3 \sin\alpha + 7/36 = 0$ 2 ✓

$\sin\alpha = \frac{24 \pm 18}{18} \Rightarrow \sin\alpha = 7/3$ غ ق ق

$\sin\alpha = 1/3$ ✓

$\sin^2 + \cos^2 = 1 \Rightarrow 1 - 1/9 = \cos^2 \Rightarrow \cos^2 = 8/9$

$1 + \tan^2 = 1/\cos^2 \Rightarrow 1 \times 9/8 = \boxed{9/8}$

-3

$\cot\left(\frac{\sin}{-\cos}\right) = \cot 3 \Rightarrow \frac{\sin}{-\cos} = 3 \Rightarrow \sin = -3\cos$

$\sin^2 + \cos^2 = 1 \Rightarrow 9\cos^2 + \cos^2 = 1 \Rightarrow \cos^2 = 1/10$

$\cos = \pm \frac{\sqrt{10}}{10} \Rightarrow \sin = 3\sqrt{10}/10$

غ ق ق ($+$)

ق ق ($-$) → ناحیه دوم داریم

2 ✓

$\sin - \cos = \frac{3}{\sqrt{10}} - \left(-\frac{1}{\sqrt{10}}\right) = \boxed{\frac{4\sqrt{10}}{10}}$

-4

$\sin^4 + 1 - \sin^2 = 4/5 \Rightarrow \sin^4 - \sin^2 + 1/5 = 0$

$(1-\cos^2)^2 + \cos^2 = 4/5 \Rightarrow \cos^4 - \cos^2 + 1/5 = 0$

$\Rightarrow \sin^4 - \sin^2 = \cos^4 - \cos^2 \Rightarrow \cos^4 + \sin^2 = \sin^4 + \cos^2$

$\Rightarrow \boxed{4/5}$

2 ✓

-5

ناحیه سوم

$\frac{\sin^2 - 1}{\cos} > 0 \Rightarrow \cos < 0$

$\frac{\sin(3+\cos)}{2 - 2\cos^2} < 0 \rightarrow \sin < 0$

$2 - 2\cos^2 = 2(1-\cos^2) = 2\sin^2$

2 ✓

-6

$\frac{2\cos + 3\sin}{\sin\cos} = 0 \rightarrow 2\cos = -3\sin \Rightarrow \cos = -3/2 \sin$

$\tan + \cot = \frac{\sin}{\cos} + \frac{\cos}{\sin} \Rightarrow \frac{\sin}{-3/2\sin} + \frac{-3/2\sin}{\sin}$

$= -2/3 - 3/2 = \boxed{-13/6}$ 2 ✓

-7

$1 + \sin 50 = 2\sin\left(\frac{90+50}{2}\right) \times \cos\left(\frac{90-50}{2}\right)$

$= 2\underbrace{\sin 70}_{\cos 20}\cos 20 = 2\cos^2 20$

$\sin 50 + \sin 10 = 2\sin\left(\frac{50+10}{2}\right) \times \cos\left(\frac{50-10}{2}\right)$

$2\sin 30 \cdot \cos 20 = \cos 20$

$\frac{\sqrt{2(\cos^2 20)}}{\cos 20} = \boxed{\sqrt{2}}$ 2 ✓

-8

$1 - 2\sin^2 5 = \cos^2 5 - \sin^2 5 = \cos 10$

$2\cos^2 20 - 1 = \cos^2 20 - \sin^2 20 = \cos 40$

$\frac{1 - \tan^2 20}{1 + \tan^2 20} = \cos 40$ 2 ✓

$\Rightarrow \sin 10 \cos 20 \cos 40 = 1/8 \frac{\sin 80}{\sin 10} = \boxed{1/8 \cot 10^\circ}$

$\times \frac{\cos 10}{\cos 10}$ ... $\frac{1}{2}\sin 20$ ... $\frac{1}{4}\sin 40$ ... $\frac{1}{4}\sin 80$

9-

$\sin\alpha = 2 - 3\cos\alpha$

$\cos^2\alpha + \underbrace{(2-3\cos\alpha)^2}_{\sin^2} = 1 \Rightarrow 10\cos^2 - 12\cos = -3$

$\cos^2 = \dfrac{1+\cos 2\alpha}{2} \Rightarrow 2\cos^2 - 1 = \cos 2\alpha$

$\Rightarrow 10\cos^2 - 5 = 5\cos 2\alpha$ 2 ✓

$\Rightarrow 10\cos^2 - 12\cos - 5 = -3 + (-5) = \boxed{-8}$

10-

$\sin\alpha \times \sin b = \frac{1}{2}(\cos(\alpha-b) - \cos(\alpha+b))$

$\Rightarrow \sin B \sin C = \frac{1}{2}(\cos(B-C) - \cos(\cancel{B}+C))$

$\cos\alpha \times \cos b = \frac{1}{2}(\cos(\alpha+b) + \cos(\alpha-b))$

$\Rightarrow \cos\frac{A}{2} \times \cos\frac{A}{2} = \frac{1}{2}(\cos A + \underbrace{\cos 0}_{1})$

$A+B+C = 180 \rightarrow \frac{1}{2}\cos(180-(2C+A)) - \cos(180-A)$

2 ✓

$\Rightarrow \frac{1}{2}(-\cos(2C+A) + \cos A))$

$-\frac{1}{2}\cos(2C+A) + \frac{1}{2}\cos A = \frac{1}{2}\cos A + \frac{1}{2}$

$\Rightarrow \cos(2C+A) = -1$

$\Rightarrow 2C + A = 180$

$B = 72 = C \Rightarrow A = 180 - 144 = \boxed{36^\circ}$