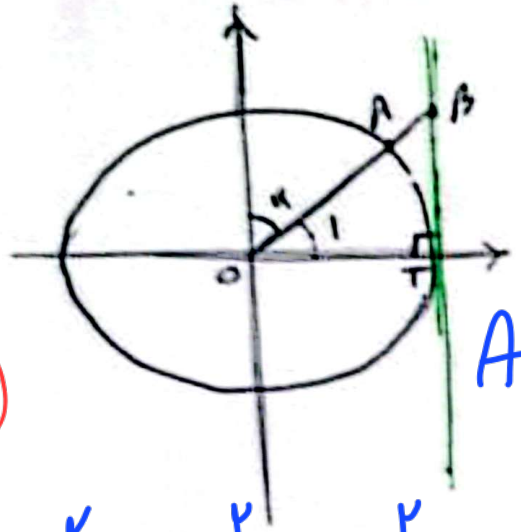
۱-



$OB = \tan\left(\frac{\pi}{2} - \alpha\right) = \cot(\alpha)$

$OA = 1$

$AB = \cot\alpha - 1 \quad \frac{\cos\alpha}{\sin\alpha} - 1 = \frac{\cos\alpha - \sin\alpha}{\sin\alpha}$

$\frac{-(\sin\alpha - \cos\alpha)}{\sin\alpha} = \left|\frac{-\sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right)}{\sin\alpha}\right|$

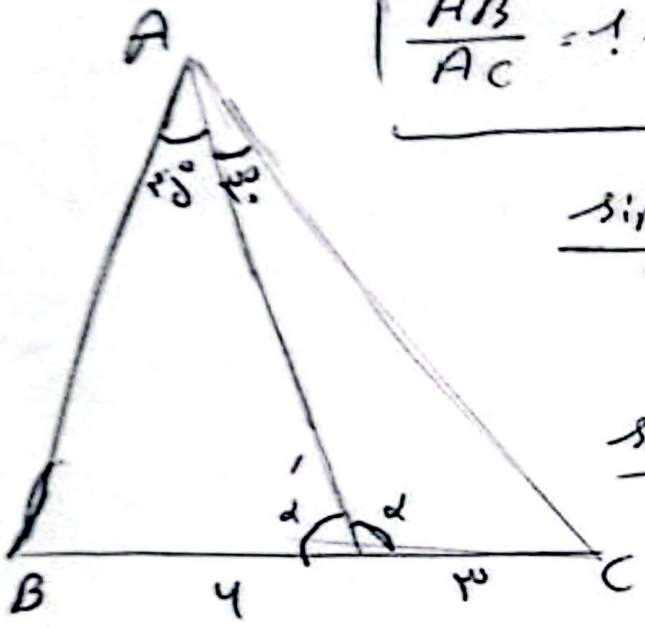
$AB = \frac{1}{\sin\alpha} - 1$

(۵)

$OB^2 = OT^2 + BT^2 \rightarrow OB^2 = 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha} \xrightarrow[OB>0]{1+\cot^2\alpha} OB = \frac{1}{\sin\alpha}$

۲-

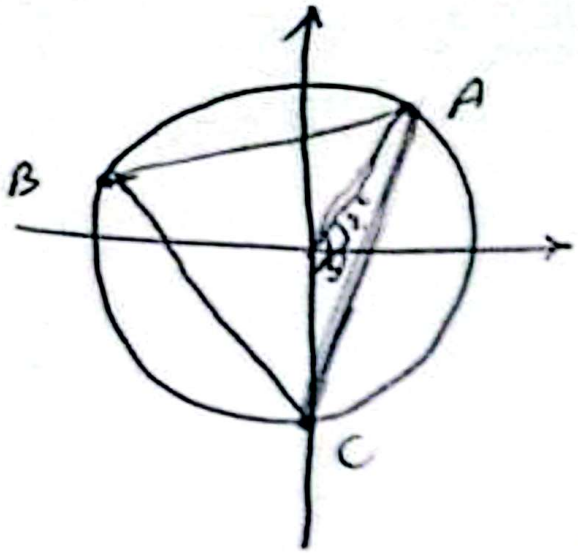
$\left|\frac{AB}{AC} = 1 \longrightarrow \frac{4\sqrt{2}\sin\alpha}{4\sin\alpha} = \sqrt{2}\right|$ ✓



(۲)

$\frac{\sin 45^\circ}{4} = \frac{\sin\alpha}{AB} \Rightarrow \frac{\sqrt{2}}{12} = \frac{\sin\alpha}{AB} \Rightarrow AB = 4\sqrt{2}\sin\alpha$

$\frac{\sin 30^\circ}{3} = \frac{\sin\alpha}{AC} \Rightarrow \frac{1}{4} = \frac{\sin\alpha}{AC} \Rightarrow AC = 4\sin\alpha$



$A\left|\frac{1}{2}\right.\left|\frac{\sqrt{3}}{2}\right.$ $B\left|\frac{-\sqrt{3}}{2}\right.\left|\frac{1}{2}\right.$ $C\left|0\right.\left|-1\right.$

$AB = \sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^2} = \sqrt{\left(\frac{4+2\sqrt{3}}{4}\right)+\left(\frac{4-2\sqrt{3}}{4}\right)} = \sqrt{2}$

$BC = \sqrt{\left(0+\frac{\sqrt{3}}{2}\right)^2+\left(-1+\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = \sqrt{1}$

$AC = \sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{4+2\sqrt{3}}{4}} = \sqrt{\frac{1+4+2\sqrt{3}}{4}} = \sqrt{1+\sqrt{3}}$

Perimeter $= \sqrt{2}+\sqrt{3}+\sqrt{1+\sqrt{3}}$ ✓

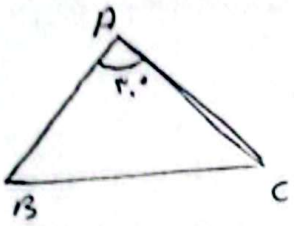
$\theta = 2B \quad \theta = 120^\circ$

$B = 60^\circ$

Area $= \frac{1}{2} \times \frac{\sqrt{2}(1+\sqrt{3})}{2} \times \sqrt{2} \times \sqrt{2} = \frac{3+\sqrt{3}}{2}$ ✓

$\sin 60^\circ = \cos 30^\circ \sin 0^\circ + \cos 0^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

(2)



$AC = \log_{\sqrt{3}} 14$

$BC = \log_{2} 27$

$S_{\Delta} = \frac{1}{2} \times \frac{1}{2} \times \log_{\sqrt{3}} 14 \times \log_{2} 27 = \frac{1}{4} \times \frac{\log 14}{\log \sqrt{3}} \times \frac{\log 27}{\log 2} = \frac{1}{4} \times \frac{4}{\log 3} \times \frac{3}{\log 2} = \frac{3}{\log 2}$

(۲)

ب-

$2\cos^4\alpha - m^2 - \sin^2\alpha(2\sin^2\alpha - 1) = 2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$\cos^2\alpha(2\cos^2\alpha - 1) - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$2\cos^2\alpha - \sin^2\alpha - \cos^2\alpha$ $\quad$ $2\sin^2\alpha - \sin^2\alpha - \cos^2\alpha$

$(\cos^2\alpha - \sin^2\alpha)$ $\quad$ $\sin^2\alpha - \cos^2\alpha$

$\cos^2 2\alpha$ $\quad$ $-(\cos^2\alpha - \sin^2\alpha)$

$\cos 2\alpha$

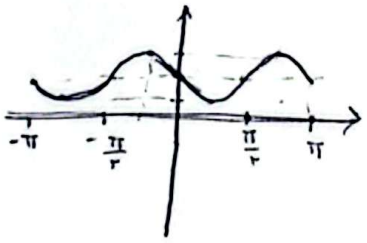
(۲)

$\cos^2 2\alpha \cos 2\alpha + \sin^2\alpha \cos 2\alpha = 1$

$\cos 2\alpha(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) = 1$ $\quad$ $\cos 2\alpha = 1$

$\sin^2 2\alpha = 1 - \cos^2 2\alpha = 0$ $\quad$ $\boxed{\sin 2\alpha = 0}$

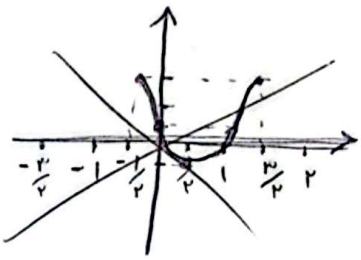
ب-۱) $y = -2\sin x \cos x + 2$ ، $[-\pi, \pi]$ $\quad$ $y = -\sin 2x + 2$ $\quad$ $T = \frac{2\pi}{2} = \pi$

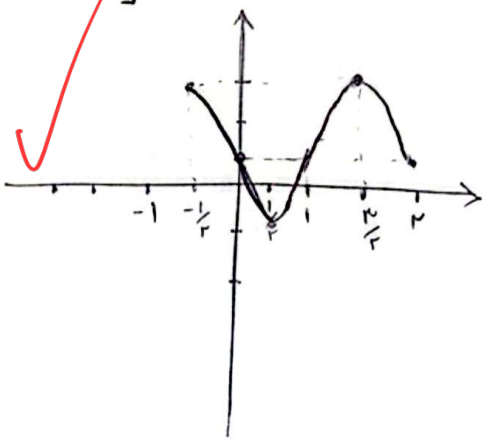


(۲)

ب) $y = 2\cos\left(x + \frac{1}{2}\right)\pi + 1$ ، $[-\pi, 2\pi] \rightarrow [-1, 2]$

$T = \frac{2\pi}{\pi} = 2$





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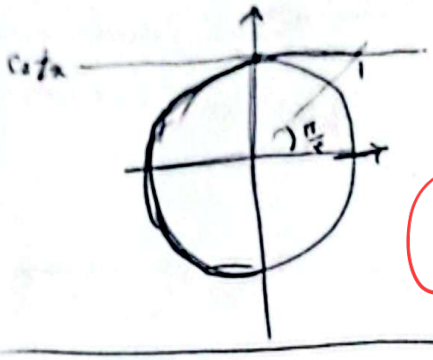
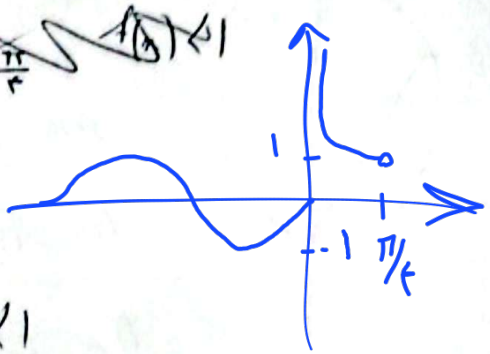
$R_f = [-1, +\infty)$

$f(x) = \begin{cases} \cot x ; & 0 < x < \frac{\pi}{2} \\ -\sin x ; & x \le 0 \end{cases}$





$-\sin x , \ x \le 0 \quad -1 \le f(x) \le 1$

$0 < f(x) < 1 \quad R_f = [-1, 1) - \{0\}$

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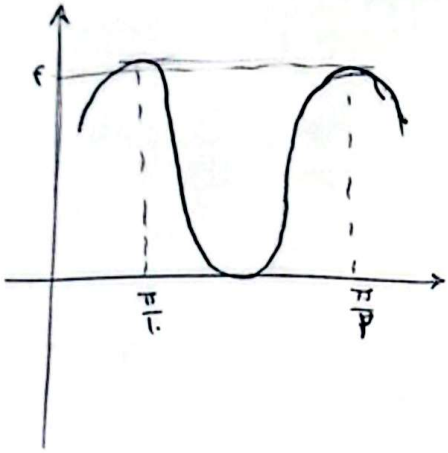
-۸

$f(x) = a + b \sin(cx) \cos(cx) \cos(2cx)$



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$a = 2$

$y = 2 + b\left(\frac{1}{2}\sin(4cx)\right) \Rightarrow 2 + \frac{b}{2}\sin(4cx)$

$b = 8$

$\sin(2c\pi) = 1 \quad c = \frac{5}{2}\sin\left(\frac{5\pi}{10}\right)$

$c = \frac{5}{4}$ $\quad \sin(2c\pi) = 1 \quad c = \frac{5}{4}\sin\left(\frac{5\pi}{2}\right)$

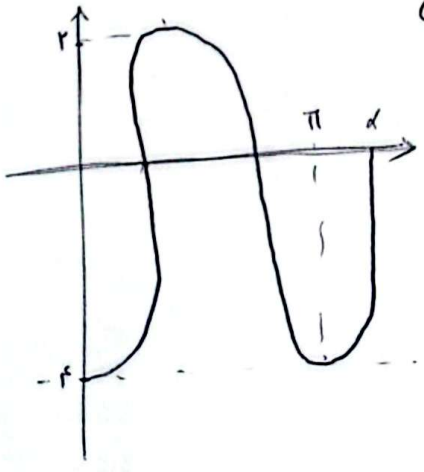
$8 \times \frac{5}{4} = 5$ ✓

-۹

$f(x) = a\cos bx + c$

$a = -3 \quad b = 2$

$c = -1$



$-3\cos 2x - 1 = 0$

$-3\cos 2\alpha = 1$

$\cos 2\alpha = -\frac{1}{3}$

۲

$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \quad \tan^2\alpha = \frac{1}{\cos^2\alpha} - 1$

$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$ ✓

$|\cos 2\alpha| = \frac{1}{3}$

$|-\sin 2\alpha| = \frac{2\sqrt{2}}{3}$

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