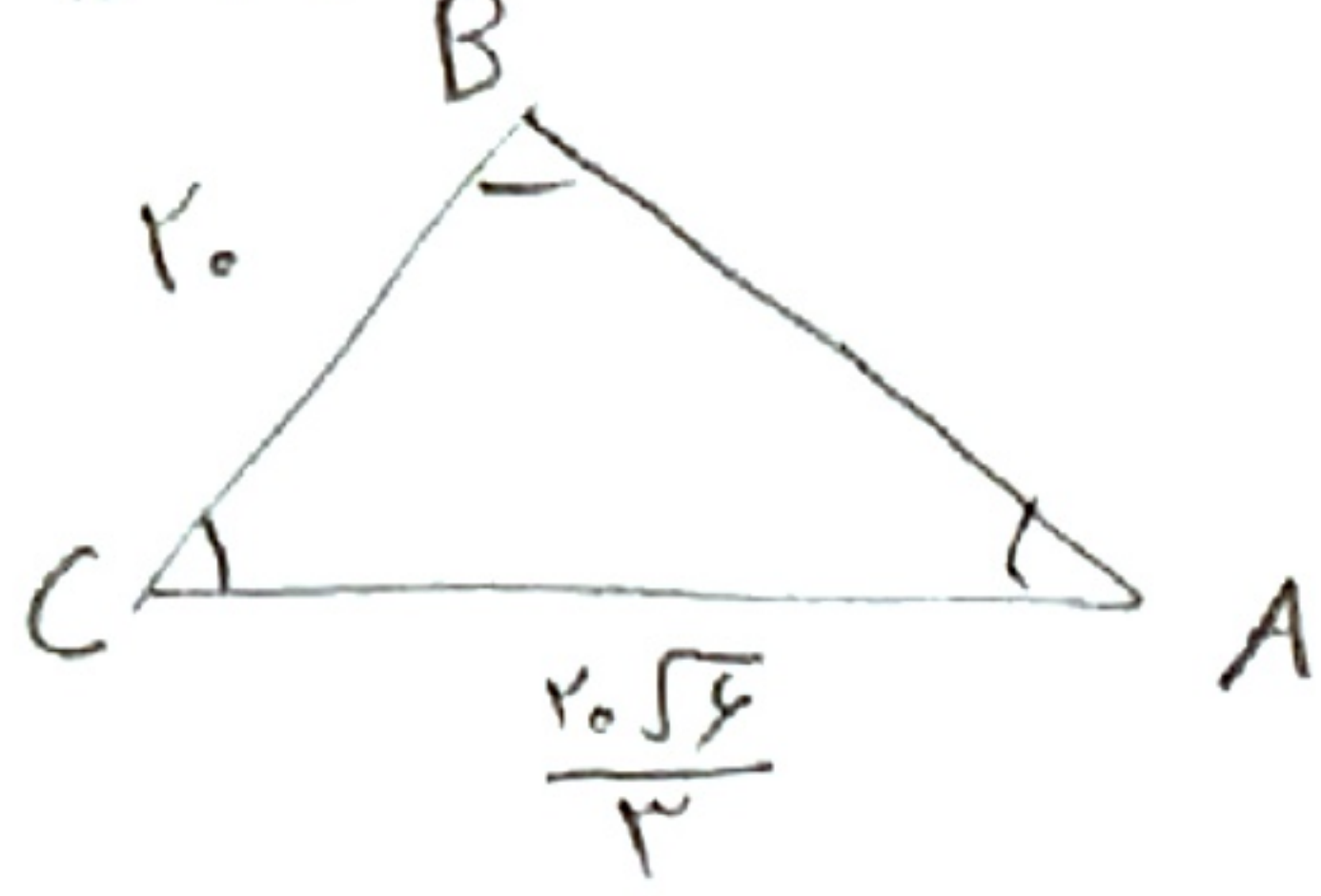


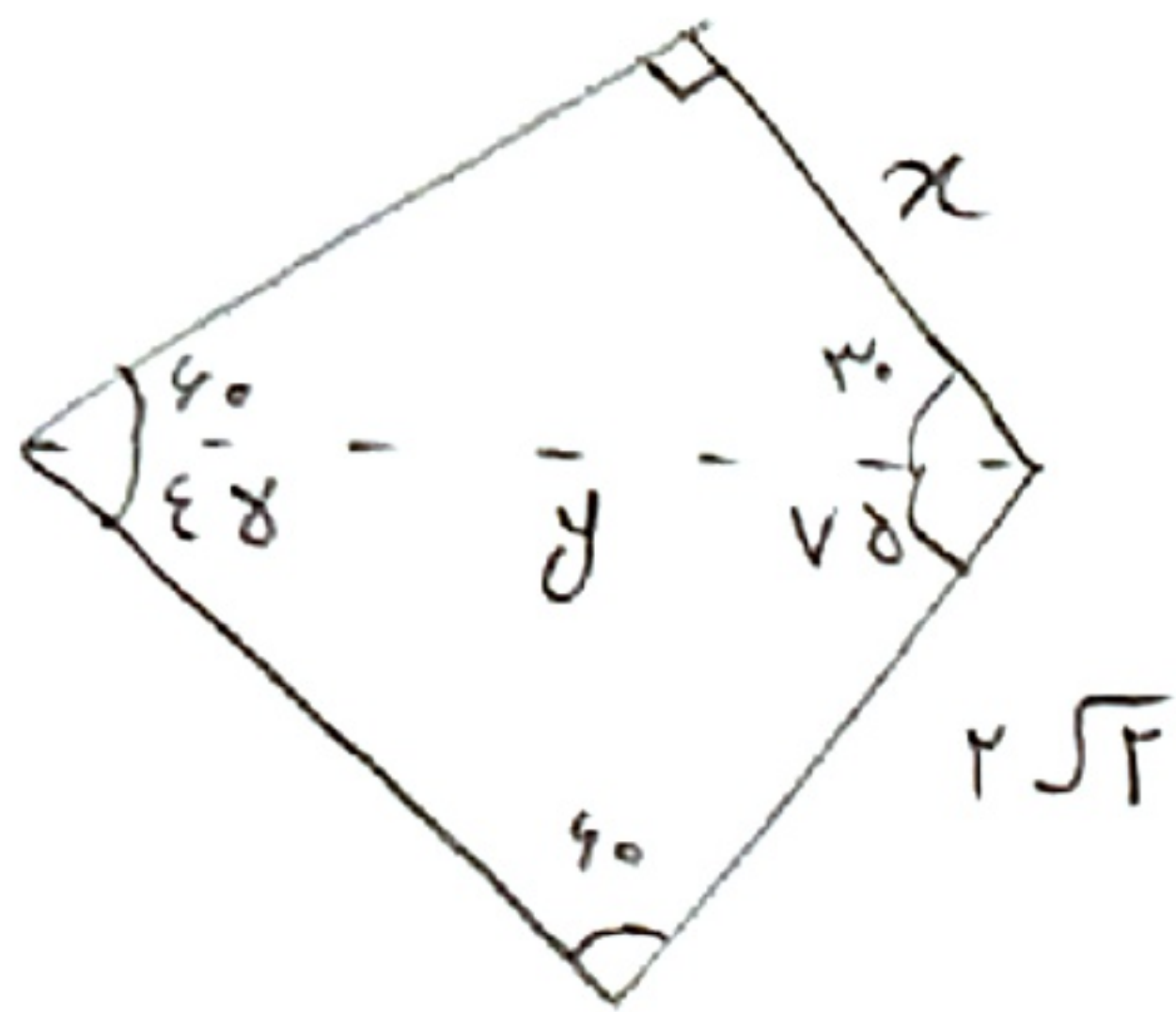
$$\hat{B} + \hat{C} = 120^\circ, \quad \hat{A} = 40^\circ$$



$$\text{قضیه سینوس ها} \rightarrow \frac{20}{\sin \hat{A}} = \frac{20 \cdot \frac{\sqrt{3}}{3}}{\sin \hat{B}} \rightarrow \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \sin \hat{B}$$

$$\sin \hat{B} = \frac{\sqrt{3}}{2} \rightarrow \hat{B} = 60^\circ, \quad \hat{C} = 80^\circ$$

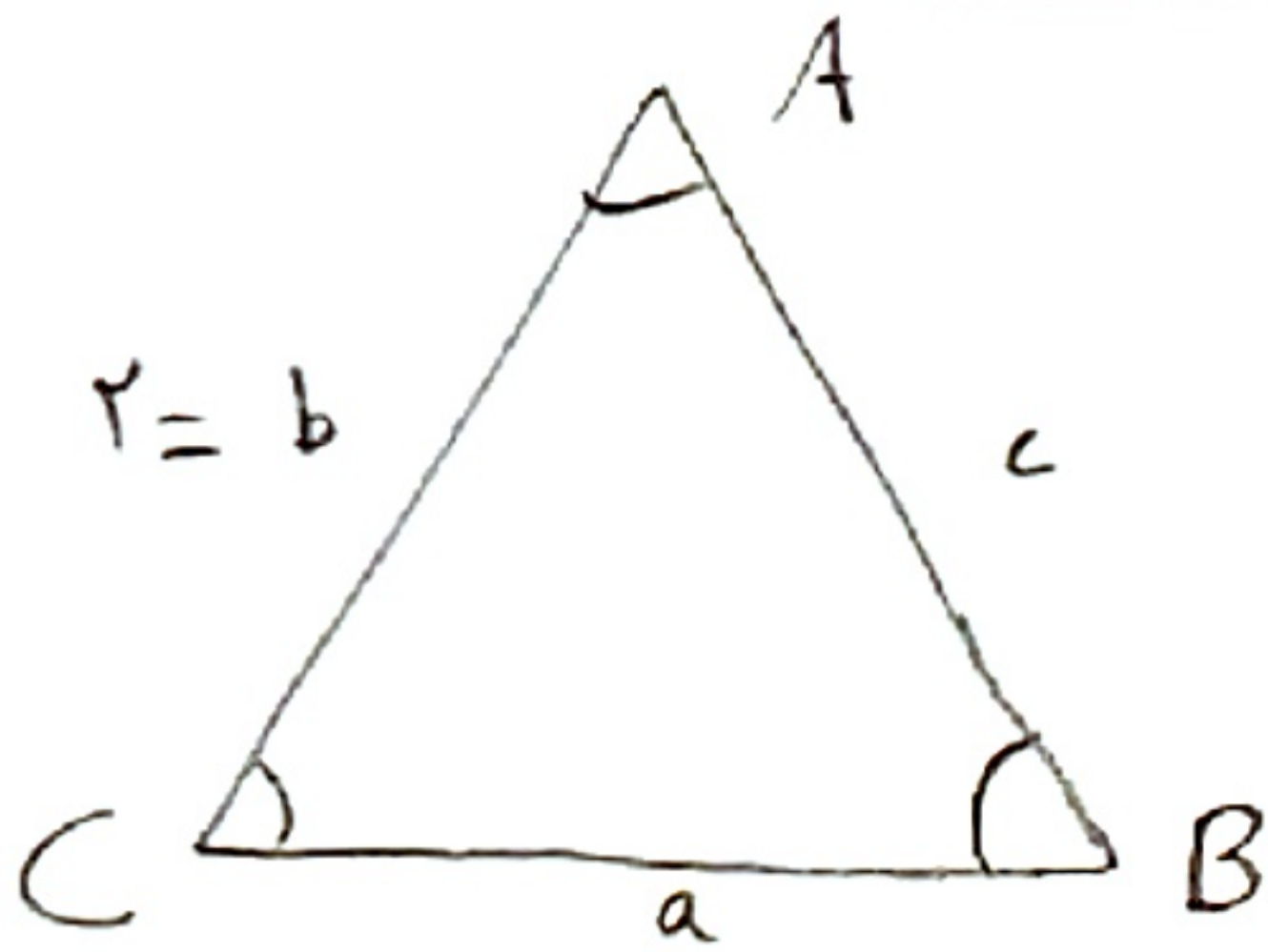
۱



$$\text{قضیه سینوس ها} \rightarrow \frac{y}{\sin 40^\circ} = \frac{2\sqrt{2}}{\sin 48^\circ} \rightarrow y = 2\sqrt{3}$$

$$\frac{y = 2\sqrt{3}}{\sin 90^\circ} = \frac{x}{\sin 90^\circ} \rightarrow x = 2$$

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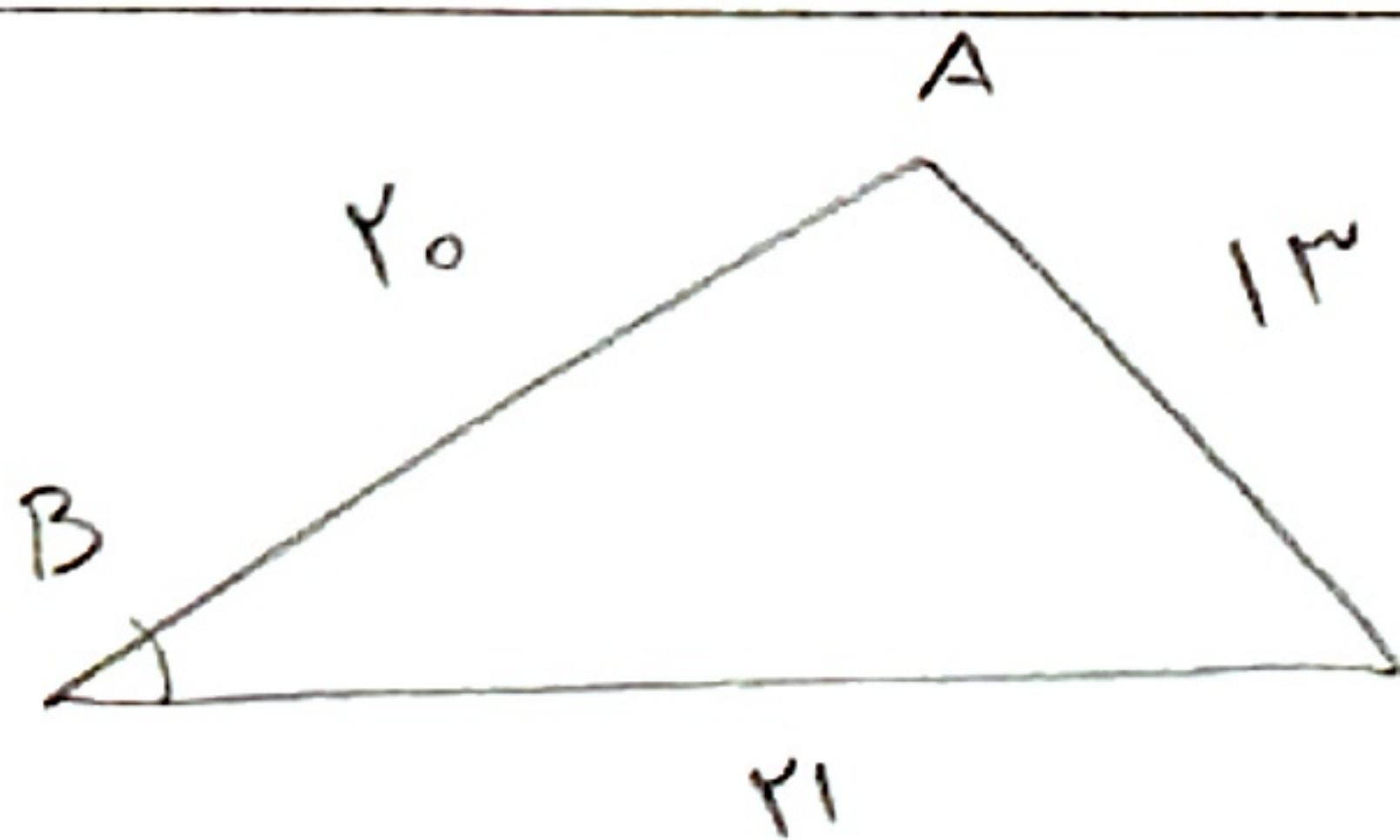


$$(b^2 - 2) \frac{\sin \hat{C}}{\sin \hat{B}} = c \rightarrow \frac{b^2 - 2}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

$$b^2 - 2 = b \rightarrow b^2 - b - 2 = 0$$

$$b = 2$$

۳

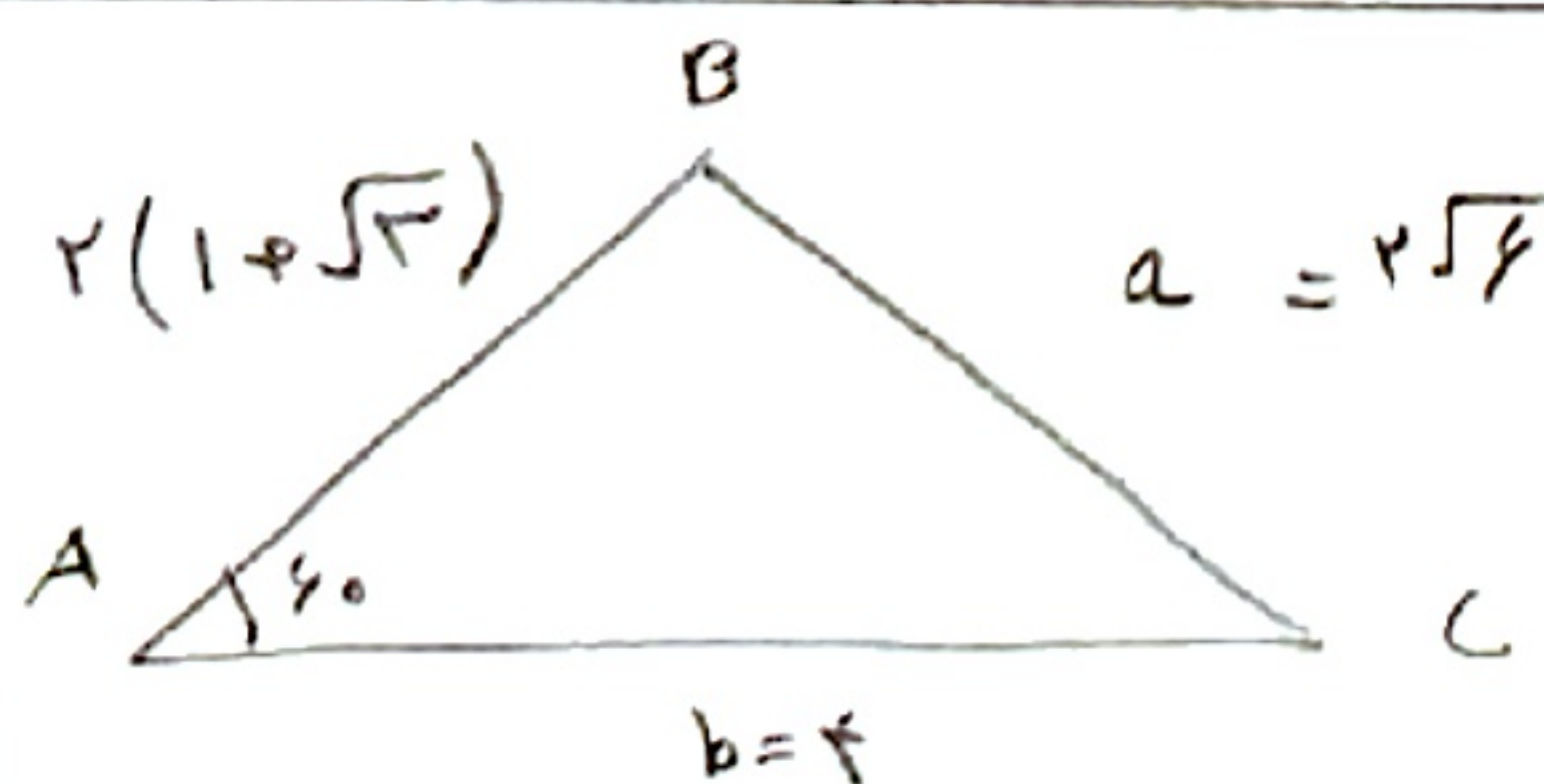


$$13^2 = 20^2 + 21^2 - 2 \times 20 \times 21 \times \cos \hat{B}$$

$$169 = 400 + 441 - 840 \cos \hat{B}$$

$$0.11 = \cos \hat{B} \rightarrow \sin \hat{B} = 0.99$$

۴



$$a = \sqrt{14 + 4(1 + \sqrt{3}) - 1(1 + \sqrt{3})} = \sqrt{12 - 1 + 1\sqrt{3} - 1\sqrt{3}}$$

$$a = \sqrt{12} = 2\sqrt{3}$$

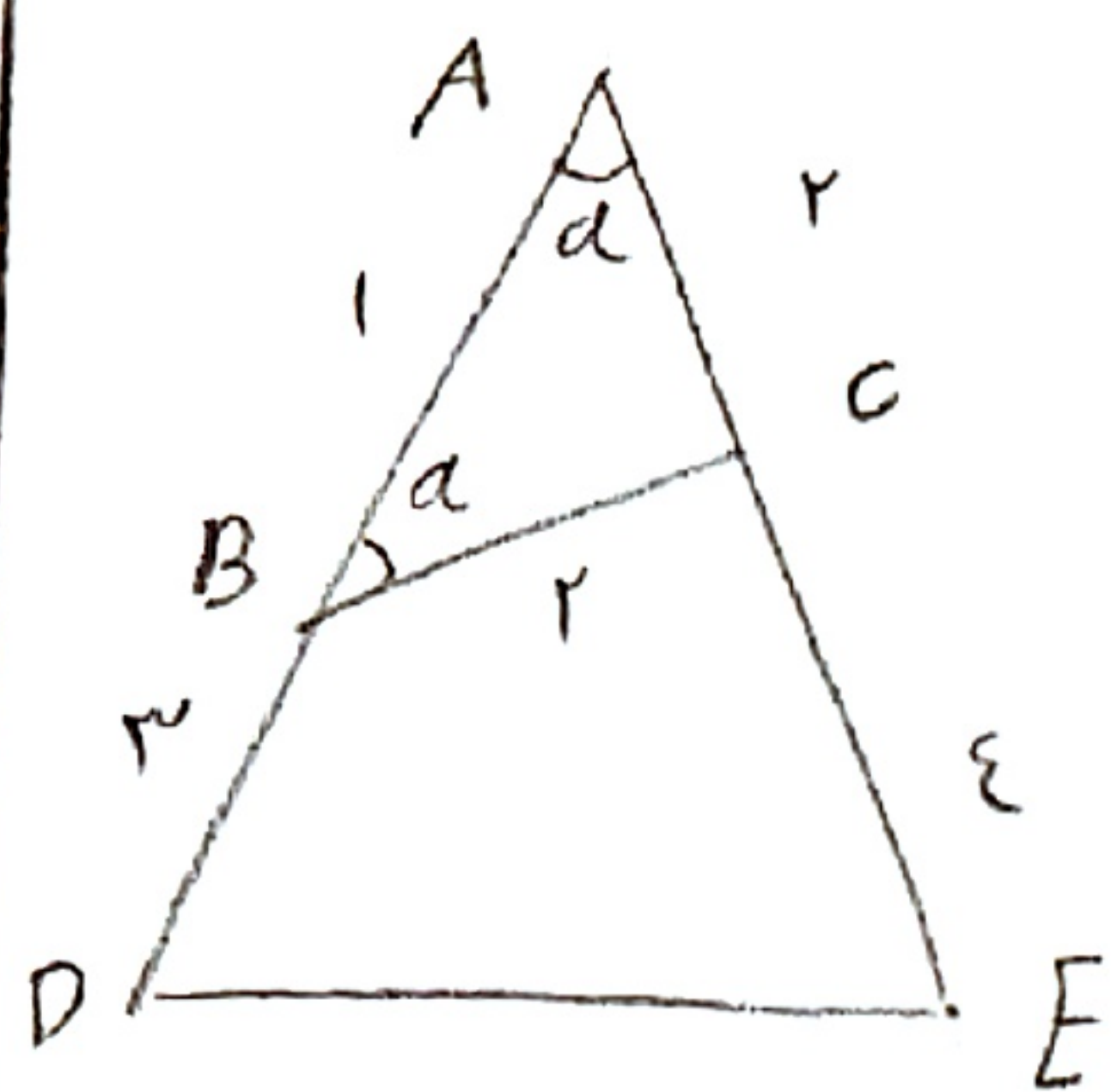
(الف)

۵

$$\text{قضیه سینوس ها} \rightarrow \frac{2\sqrt{6}}{\sin 40^\circ} = \frac{2}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{\sqrt{6}}{2}, \quad \hat{B} = 48^\circ$$

$$\hat{C} = 88^\circ$$

ب

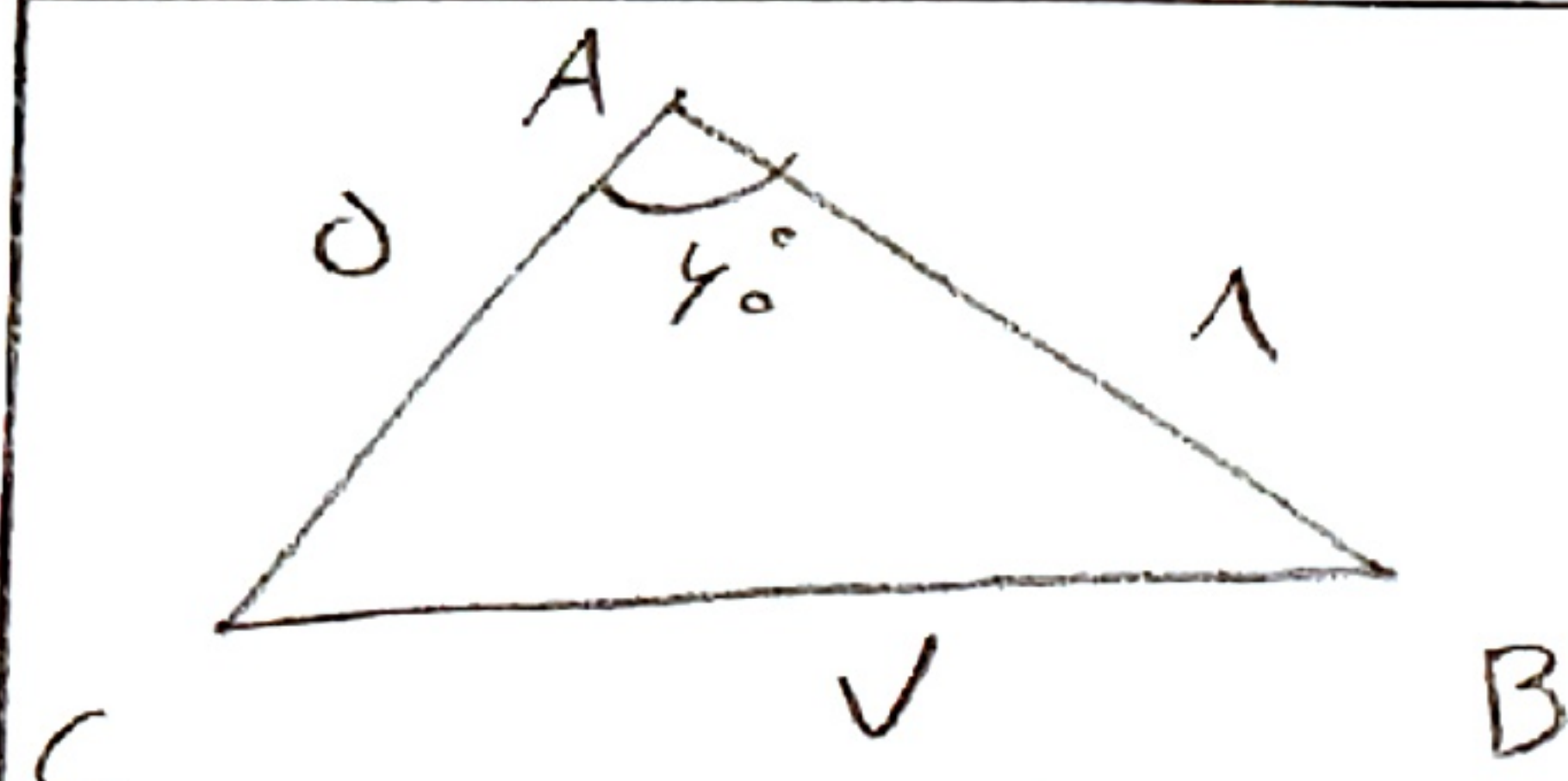


$$AC = \sqrt{1+r^2 - r^2 \cos \alpha} = r$$

$$r = 1 - r \cos \alpha \rightarrow \cos \alpha = \frac{1}{r}$$

$$DE = \sqrt{r^2 + 1^2 - r \cdot 1 \cdot \left(\frac{1}{r}\right)} = \sqrt{r^2 + 1 - 1} = r$$

6



$$CB = \sqrt{r^2 + 1^2 - 1 \cdot r \cdot \left(\frac{1}{r}\right)} = V$$

$$BC = V$$

(w)

v

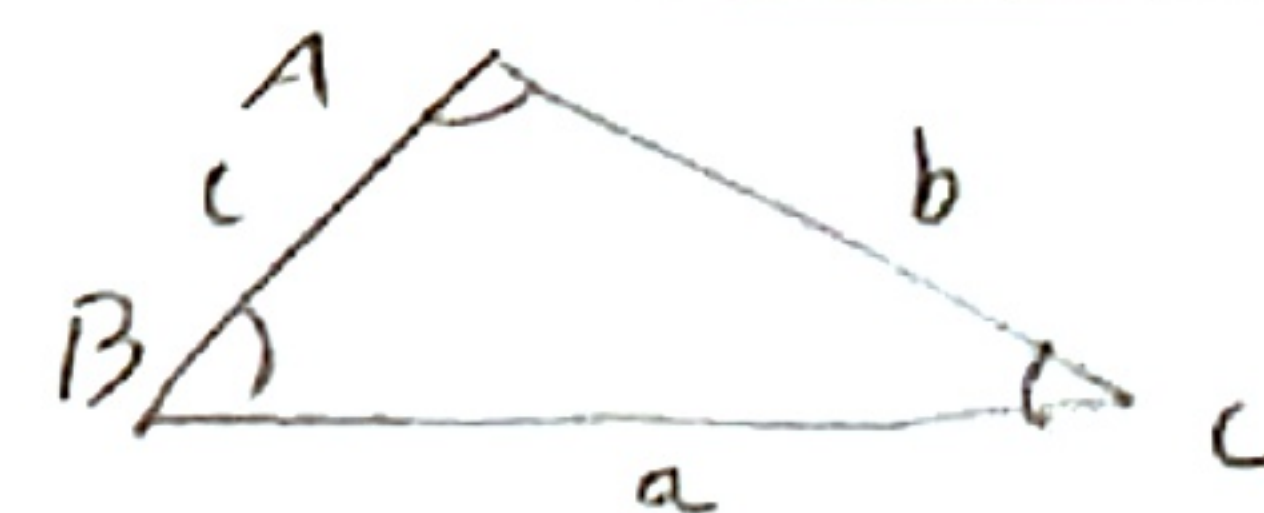
$$S = \frac{1}{r} \times AC \times AB \times \sin A = \frac{1}{r} \times 8 \times 1 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

(c)

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)}$$

$$\rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc \cos A = b^2 + c^2 - a^2$$



A

$$\downarrow$$

$$\frac{b^2 + c^2 - a^2}{bc(\cos A + 1)}$$

$$= \frac{2bc \cos A + bc}{bc(\cos A + 1)} = \frac{2bc(\cos A + 1)}{bc(\cos A + 1)} = 2$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

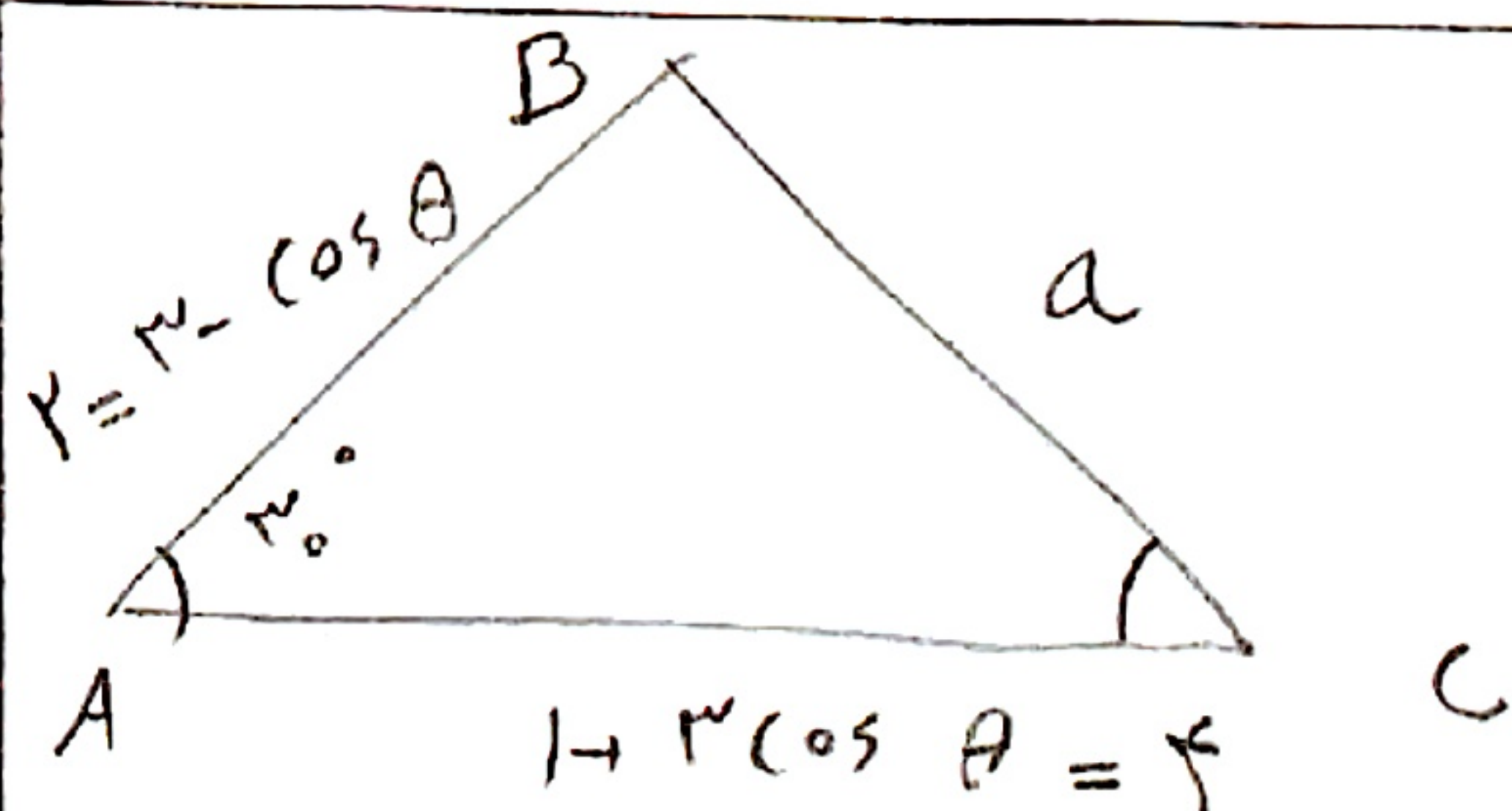
$$a^2 + b^2 - c^2 = a^2(a+b-c)$$

$$a^2 + b^2 - c^2 = a^2 + a^2b - a^2c \rightarrow b^2 - c^2 = a^2(b-c)$$

$$(b-c)(b+c+a) = a^2(b-c)$$

$$b+c+a = a^2 \rightarrow -2 \cos A = -1 \rightarrow \cos A = \frac{1}{2}$$

9



$$S = \frac{1}{r} \sin \theta (r \cos \theta) (1 + r \cos \theta) = r$$

$$r \cos \theta (1 + r \cos \theta) = 1 \rightarrow \cos \theta = 1$$

$$a^2 = r^2 + 1^2 - 1 \cdot r \cdot \cos \theta = \frac{5r^2}{r}$$

$$a^2 = r^2 - 1$$

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