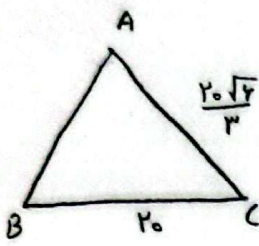


-۱



$$\hat{B} + \hat{C} = 120^\circ, \hat{B}, \hat{C} = ?$$

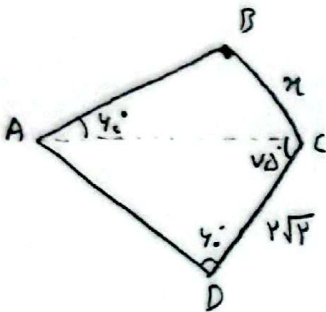
$$\Delta ABC: \hat{A} + \hat{B} + \hat{C} = 180^\circ \rightarrow \hat{A} = 60^\circ$$

قضیه سینوس ها: $\frac{BC}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}} \rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{20\sqrt{2}/3}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ$

$\hat{B} + \hat{C} = 120^\circ \rightarrow \hat{C} = 75^\circ$ ✓

(۲)

-۲



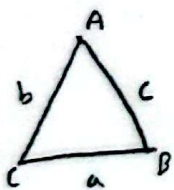
$$\Delta ADC: \hat{A}_1 + \hat{D} + \hat{C}_1 = 180^\circ \rightarrow \hat{A}_1 + 70 + 40 = 180^\circ \rightarrow \hat{A}_1 = 70^\circ$$

قضیه سینوس ها: $\frac{DC}{\sin \hat{A}_1} = \frac{AC}{\sin \hat{D}} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{\sqrt{2}}{2}} \rightarrow AC = 2\sqrt{3}$

$$\Delta ABC: \hat{B} = 70^\circ \rightarrow \sin 70^\circ = \frac{x}{AC} \rightarrow \frac{\sqrt{3}}{2} = \frac{x}{2\sqrt{3}} \rightarrow x = 3$$

(۲)

-۳

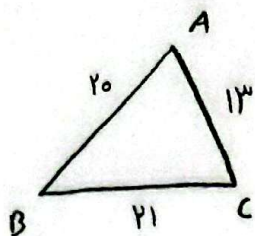


$$(b^2 - r) \frac{\sin \hat{C}}{\sin \hat{B}} = c$$

قضیه سینوس ها: $\frac{c}{\sin \hat{C}} = \frac{b}{\sin \hat{B}} \rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b}$ ①

$$(b^2 - r) \frac{\sin \hat{C}}{\sin \hat{B}} = c \xrightarrow{①} (b^2 - r) \frac{c}{b} = c \rightarrow b - \frac{r}{b} = 1 \rightarrow b = 2$$

(۲)

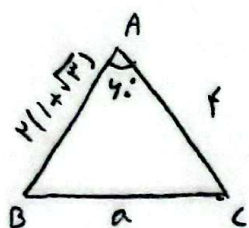
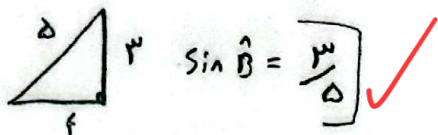


$$\sin \hat{B} = ?$$

قفسی کسینوس ها

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos \hat{B}$$

$$\rightarrow 169 = 400 + 441 - 2 \times 20 \times 21 \times \cos \hat{B} \rightarrow \cos \hat{B} = \frac{f}{\Delta}$$



قفسی کسینوس ها

$$a^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \hat{A}$$

$$\rightarrow a^2 = 16 + 4\sqrt{3} + 4 - 2 \times 2 \times (1 + \sqrt{3}) \times \frac{1}{2}$$

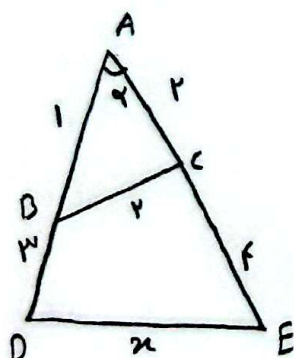
$$\rightarrow a^2 = 2f \rightarrow a = \sqrt{2f} \rightarrow a = 2\sqrt{f}$$

الف) اندازه ی ضلع a را بیابید.

ب) زاویه های B و C مشخص کنید.

قفسی سینوس ها

$$\frac{a}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{f}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ, \hat{C} = 45^\circ$$



$$x = ?$$

قفسی کسینوس ها

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \hat{A}$$

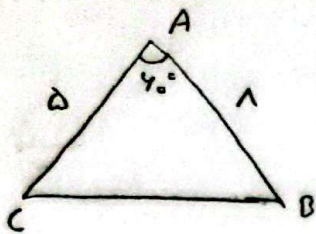
$$\rightarrow f = 1 + 1 - 2 \times 1 \times 2 \times \cos \hat{A} \rightarrow \cos \hat{A} = \frac{1}{f}$$

قفسی کسینوس ها در مثلث ADE

$$DE^2 = AD^2 + AE^2 - 2 \times AD \times AE \times \cos \hat{A}$$

$$\rightarrow DE^2 = 1^2 + 4^2 - 2 \times 1 \times 4 \times \frac{1}{f} \rightarrow DE^2 = f_0$$

$$\rightarrow DE = x = 2\sqrt{f_0}$$



الف) طول ضلع BC = ؟

قانون جيبس : $BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \times \cos \hat{A}$

$\rightarrow BC^2 = 2^2 + 1^2 - 2 \times 2 \times 1 \times \frac{1}{2} \rightarrow BC^2 = 1 \rightarrow BC = 1$

$S_{\triangle ABC} = \frac{1}{2} \times AC \times AB \times \sin \hat{A} = \frac{1}{2} \times 2 \times 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

ب) مساحت مثلث ABC

$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = ?$

قانون جيبس : $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$ ①

① $\frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = 2$

$\frac{a^2 + b^2 - c^2}{a+b-c} = a$ طرفين $\rightarrow a^2 + b^2 - c^2 = a^2 + a^2b - a^2c \rightarrow b^2 - c^2 = a^2b - a^2c$

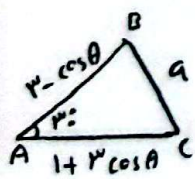
$\cos \hat{A} = ?$

$\rightarrow (b-c)(b+c) = a^2(b-c) \rightarrow a^2 = b^2 + c^2 + bc$ ①

قانون جيبس : $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$ ②

① - ② $-2bc \cos \hat{A} = bc$

$\rightarrow \cos \hat{A} = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$



$S_{\triangle ABC} = \frac{1}{2} \times AB \times AC \times \sin \theta \rightarrow r = \frac{1}{2} \times (r \cos \theta)(1 + r \cos \theta) \rightarrow \cos \theta = 1$

قانون جيبس : $a^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \hat{A}$

$a^2 = r^2 + 1^2 - 2 \times r \times 1 \times \frac{r}{r} \rightarrow a^2 = r^2 - 1 \rightarrow a = \sqrt{r^2 - 1}$