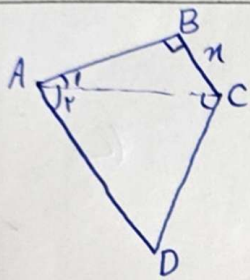


$$\hat{B} + \hat{C} = 120^\circ \quad \hat{A} + \hat{B} + \hat{C} = 180^\circ \quad \hat{A} = 60^\circ \quad \frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$$

$$\Rightarrow \frac{10}{\frac{\sqrt{3}}{2}} = \frac{10\sqrt{2}}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow \underline{\hat{B} = 45^\circ} \Rightarrow \underline{\hat{C} = (120^\circ - 45^\circ) = 75^\circ}$$



$$\hat{A}_r = 180^\circ - 75^\circ - 40^\circ = 65^\circ, \quad \frac{DC}{\sin \hat{A}_r} = \frac{AC}{\sin \hat{D}} \Rightarrow \frac{x\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow AC = 2\sqrt{3}$$

$$x = \sin 40^\circ \times AC = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = \underline{3}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} = \frac{c}{b-r} \Rightarrow b = b-r \Rightarrow b-b-r=0$$

$$\Rightarrow (b-r)(b+r)=0 \quad \begin{cases} b=-1 \quad \times \\ \underline{b=r} \quad \checkmark \end{cases}$$

$$b^2 = a^2 + c^2 - 2ac \cos \hat{B} \Rightarrow 13^2 = 10^2 + 11^2 - 2 \times 10 \times 11 \times \cos \hat{B} \Rightarrow 140 \cos \hat{B} = 44$$

$$\Rightarrow \cos \hat{B} = \frac{44}{140} = 0.314 \rightarrow \sin \hat{B} = 0.94$$

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow a^2 = 14 + (2+2\sqrt{3})^2 - 2 \times (2+2\sqrt{3}) \times 4 \times \frac{1}{2}$$

$$a^2 = 14 + 4 + 12\sqrt{3} + 12 - 4 - 4\sqrt{3} = 24 \rightarrow \underline{a = 2\sqrt{6}}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{2}}{2}} = \frac{r}{\sin B} = \frac{2+2\sqrt{3}}{\sin C} \Rightarrow \sin \hat{B} = \frac{2\sqrt{3}}{2\sqrt{6}} = \frac{\sqrt{2}}{2} \quad (ب)$$

$$\Rightarrow \underline{\hat{B} = 45^\circ}, \quad \underline{\hat{C} = 75^\circ}$$

$$\cos \alpha = \frac{AB^2 + AC^2 - BC^2}{2AB \times AC} = \frac{1 + 4 - 4}{2 \times 1 \times 1} = \frac{1}{2}, DE^2 = AD^2 + AE^2 - 2 \times AD \times AE \cos \alpha$$

$$= 1 + 4 - 2 \times 1 \times 2 \times \frac{1}{2} = 4 \Rightarrow DE = \sqrt{4} = 2$$

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$$CB^2 = AC^2 + AB^2 - 2AB \cdot AC \cos 40^\circ \Rightarrow CB^2 = 25 + 49 - 2 \times 5 \times 7 \times \frac{1}{2} = 49$$

$$\Rightarrow CB = 7$$

ب

$$S = \frac{1}{2} \times 5 \times 7 \times \sin 40^\circ = 10 \times \frac{\sqrt{10}}{2} = 10\sqrt{10}$$

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$$\frac{b^2 + c^2 + 2bc - a^2}{bc \cos A + bc} = \frac{b^2 + c^2 - a^2}{bc(\cos A + 1)} + \frac{2bc}{bc(\cos A + 1)} = (\cos A + 1) \frac{1}{\cos A + 1}$$

$$= \frac{\cos A + 1}{\cos A + 1} = 1$$

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$$a^2 + a^2b - a^2c = a^2 + b^2 - c^2 \Rightarrow a^2(b - c) = b^2 - c^2$$

$$\Rightarrow (b - c)(b^2 + bc + c^2) = a^2(b - c)$$

$$\Rightarrow a^2 = b^2 + c^2 + bc$$

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$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{b^2 + c^2 - b^2 - c^2 - bc}{2bc} = -\frac{1}{2}$$

$$S = \frac{1}{2} ab \sin 30^\circ = 1 = \frac{1}{2} (3 - \cos \theta)(1 + 3 \cos \theta) \times \frac{1}{2}$$

$$\Rightarrow 3 \cos^2 \theta - 1 \cos \theta + 1 = 0 \Rightarrow (3 \cos \theta - 1)(\cos \theta - 1) = 0$$

$\cos \theta = +1 \checkmark$
 $\cos \theta = \frac{1}{3} \times$

$$\Rightarrow a^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \cos 30^\circ = 9 + 16 - 12\sqrt{3} = 25 - 12\sqrt{3}$$

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