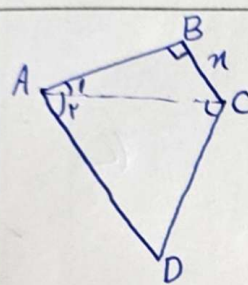


$$\hat{B} + \hat{C} = 120^\circ \quad \hat{A} + \hat{B} + \hat{C} = 180^\circ \quad \hat{A} = 60^\circ \quad \frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$$

$$\Rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{20\sqrt{3}}{\sin B} \Rightarrow \sin B = \frac{\sqrt{3}}{2} \Rightarrow \underline{\hat{B} = 60^\circ} \Rightarrow \underline{\hat{C} = (120^\circ - 60^\circ) = 60^\circ}$$

(۲)



$$\hat{A}_r = 120^\circ - 60^\circ - 60^\circ = 0^\circ, \quad \frac{DC}{\sin \hat{A}_r} = \frac{AC}{\sin \hat{D}} \Rightarrow \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow AC = 20\sqrt{3}$$

$$n = \sin 60^\circ \times AC = \frac{\sqrt{3}}{2} \times 20\sqrt{3} = \underline{30}$$

(۲)

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} = \frac{c}{b-r} \Rightarrow b = b-r \Rightarrow b-b-r=0$$

$$\Rightarrow (b-r)(b+r)=0 \Rightarrow \begin{cases} b=-r \text{ X} \\ \underline{b=r} \end{cases}$$

(۲)

$$b^2 = a^2 + c^2 - 2ac \cos \hat{B} \Rightarrow 13^2 = 10^2 + 11^2 - 2 \times 10 \times 11 \times \cos \hat{B} \Rightarrow 169 = 100 + 121 - 220 \cos \hat{B} \Rightarrow 169 = 221 - 220 \cos \hat{B}$$

$$\Rightarrow \cos \hat{B} = \frac{221 - 169}{220} = \frac{52}{220} = \frac{13}{55} \Rightarrow \underline{\sin \hat{B} = \frac{4\sqrt{21}}{55}}$$

(۲)

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow a^2 = 14^2 + (2+2\sqrt{3})^2 - 2 \times (2+2\sqrt{3}) \times 14 \times \frac{1}{2}$$

$$a^2 = 196 + 4 + 12\sqrt{3} + 12 - 14(2+2\sqrt{3}) = 196 + 16 + 12\sqrt{3} - 28 - 28\sqrt{3} = 184 - 16\sqrt{3}$$

$$\Rightarrow \underline{a = 2\sqrt{46}}$$

(الف)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{2\sqrt{46}}{\frac{\sqrt{2}}{2}} = \frac{r}{\sin B} = \frac{2+2\sqrt{3}}{\sin C} \Rightarrow \sin \hat{B} = \frac{2\sqrt{23}}{2\sqrt{46}} = \frac{\sqrt{2}}{2}$$

(ب)

$$\Rightarrow \underline{\hat{B} = 45^\circ}, \quad \underline{\hat{C} = 45^\circ}$$

(۲)

$$\cos \alpha = \frac{AB^2 + AC^2 - BC^2}{2AB \times AC} = \frac{1 + 4 - 4}{2 \times 1 \times 1} = \frac{1}{2} \checkmark, DE^2 = AD^2 + AE^2 - 2 \times AD \times AE \cos \alpha$$

$$= 1 + 4 - 2 \times 1 \times 1 \times \frac{1}{2} = 4 \Rightarrow DE = \sqrt{4} = 2 \checkmark$$

(2)

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$$CB^2 = AC^2 + AB^2 - 2AB \cdot AC \cos 90^\circ \Rightarrow CB^2 = 4 + 4 - 2 \times 2 \times 2 \times \frac{1}{2} = 4 \text{ (الق)}$$

$$\Rightarrow CB = 2 \checkmark$$

$$S = \frac{1}{2} \times 2 \times 2 \times \sin 90^\circ = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2} \checkmark$$

(2)

(ب)

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$$\frac{b^2 + c^2 + 2bc - a^2}{bc \cos A + bc} = \frac{b^2 + c^2 - a^2}{bc(\cos A + 1)} + \frac{2bc}{bc(\cos A + 1)} = (2 \cos A + 1) \frac{1}{\cos A + 1}$$

$$= \frac{2 \cos A + 1}{\cos A + 1} = 2 \checkmark$$

(2)

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$$a^2 + a^2 b - a^2 c = a^2 + b^2 - c^2 \Rightarrow a^2(b - c) = b^2 - c^2$$

$$\Rightarrow (b - c)(b^2 + bc + c^2) = a^2(b - c)$$

$$\Rightarrow a^2 = b^2 + c^2 + bc$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{b^2 + c^2 - b^2 - c^2 - bc}{2bc} = -\frac{1}{2} \rightarrow A = 120^\circ$$

(2)

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$$S = \frac{1}{2} ab \sin 120^\circ = 2 = \frac{1}{2} (2 - \cos \theta)(1 + 2 \cos \theta) \times \frac{1}{2}$$

$$\Rightarrow 2 \cos^2 \theta - 1 \cos \theta + 2 = 0 \xrightarrow{\text{نقل}} (\cos \theta - 2)(\cos \theta + 1) = 0$$

$$\Rightarrow a^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 120^\circ = 4 + 16 - 16 \times \left(-\frac{1}{2}\right) = 20 \checkmark$$

$\cos \theta = +1 \checkmark$
 $\cos \theta = -\frac{1}{2} \times$

(2)

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