

۱۳) ۲.۴

شکل ۲

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \Rightarrow \frac{20}{\frac{\sqrt{2}}{2}} = \frac{20\sqrt{2}}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{1}{\sqrt{2}} \Rightarrow \hat{B} = 45^\circ \quad \left(\hat{B} + \hat{C} = 135^\circ \Rightarrow \hat{A} = 45^\circ \right)$$

$$\Rightarrow \hat{B} + \hat{C} = 135^\circ = 90^\circ + \hat{C} \Rightarrow \hat{C} = 45^\circ$$

۱۴) $\hat{C} = 70^\circ, \hat{D} = 40^\circ \Rightarrow \hat{A}_1 = \hat{CAD} = 110^\circ = (90^\circ + 20^\circ) = 110^\circ$

قضیه سینوس: $\frac{CD}{\sin \hat{A}_1} = \frac{AC}{\sin \hat{D}} \Rightarrow \frac{2\sqrt{2}}{\sin 110^\circ} = \frac{AC}{\sin 40^\circ} \Rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{2}}{2}} \Rightarrow AC = 2\sqrt{2}$

تقریب سینوس: $\sin \hat{BAC} = \frac{BC}{AC} \Rightarrow \sin 40^\circ = \frac{\sqrt{2}}{2} = \frac{x}{2\sqrt{2}} \Rightarrow x = 2$

۱۵) قضیه سینوس: $\frac{c}{\sin \hat{C}} = \frac{b}{\sin \hat{B}} \Rightarrow \frac{c \sin \hat{B}}{\sin \hat{C}} = b$

$$\left. \begin{aligned} &\Rightarrow b = b^2 - 2 \Rightarrow b^2 - b - 2 = 0 \\ &\Rightarrow (b-2)(b+1) = 0 \Rightarrow b = 2 \end{aligned} \right\} \begin{aligned} &b = -1 \\ &\text{غیر ممکن} \end{aligned}$$

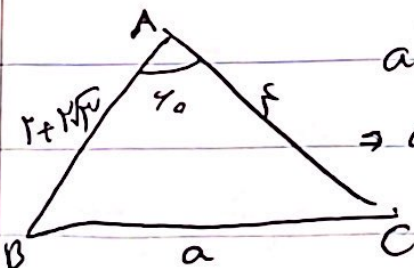
(نتیجه مثبت بماند)

۱۶) قضیه کسینوس: $b^2 = a^2 + c^2 - 2ac \cos \hat{B} \Rightarrow 13^2 = 21^2 + 20^2 - 2(21)(20) \cos \hat{B}$

$$\Rightarrow 169 = 441 + 400 - 840 \cos \hat{B} \Rightarrow \cos \hat{B} = \frac{1}{2} \Rightarrow \sin^2 \hat{B} = 1 - \cos^2 \hat{B} = 1 - \frac{1}{4} = \frac{3}{4}$$

$\Rightarrow \sin \hat{B} = \frac{\sqrt{3}}{2}$ (چون $\sin^2 \hat{A} + \cos^2 \hat{A} = 1$)

پس $\hat{B} = 60^\circ$ یا 120° (اما 120° در این مسئله ممکن نیست)



۱۷) $a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow a^2 = 4^2 + (2 + 2\sqrt{2})^2 - 2(4)(2 + 2\sqrt{2}) \cos 45^\circ$

$$\Rightarrow a^2 = 16 + 16 - 8\sqrt{2} - 8 - 8\sqrt{2} \Rightarrow a^2 = 24 \Rightarrow a = 2\sqrt{6}$$

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \Rightarrow \frac{2\sqrt{2}}{\sin 45^\circ} = \frac{2}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{1}{\sqrt{2}} \Rightarrow \hat{B} = 45^\circ \quad \text{---}$$

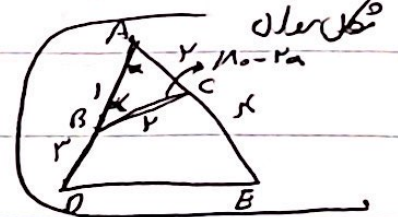
$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow 45^\circ + 45^\circ + \hat{C} = 180^\circ \Rightarrow \hat{C} = 90^\circ \quad \text{---}$$

قضیه سینوس : $\frac{AC}{\sin(\hat{A}BC)} = \frac{AB}{\sin(\hat{A}CB)}$ (BC = AC) = الی قیاسی $\hat{A}BC = \hat{A}CB$ (7)

در این حالت $\hat{A}CB = 180^\circ - \alpha$ و $\alpha = \hat{C}AB = \hat{A}BC$

$$\Rightarrow \frac{2}{\sin \alpha} = \frac{1}{\sin \alpha}$$

$$\Rightarrow \frac{2}{\sin \alpha} = \frac{1}{2 \sin \alpha \cos \alpha} \Rightarrow \cos \alpha = \frac{1}{2} \Rightarrow \cos \hat{A} = \frac{1}{2}$$



قضیه کسینوس $\Rightarrow DE^2 = AD^2 + AE^2 - 2 \cdot AD \cdot AE \cdot \cos \hat{A} \Rightarrow DE^2 = 1^2 + 2^2 - 2 \cdot 1 \cdot 2 \cdot \left(\frac{1}{2}\right) = 4$
 $\Rightarrow DE = 2$

قضیه کسینوس : $BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos 90^\circ = 4 + 2 - 0 = 6 \Rightarrow BC = \sqrt{6}$ (8)

$$S_{\triangle ABC} = \frac{1}{2} \cdot AB \cdot AC \cdot \sin 90^\circ = 1 \cdot \sqrt{2} = \sqrt{2} \quad \text{---}$$

قضیه کسینوس : $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$ (9)

$$\frac{(b^2 + c^2) - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos \hat{A})}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = 2$$

$$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2 \Rightarrow a^3 + b^3 - c^3 = (a + b - c)a^2 = a^3 + a^2(b - c)$$
 (9)

$$\Rightarrow b^3 - c^3 = a^2(b - c) \Rightarrow (b - c)(b^2 + bc + c^2) = a^2(b - c) \Rightarrow b^2 + bc + c^2 = a^2$$

قضیه کسینوس : $a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow b^2 + bc + c^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow \cos \hat{A} = -\frac{1}{2}$

$$S \Rightarrow S = \frac{1}{p} (1 + r \cos \theta) (r - \cos \theta) \sin \theta = \frac{-r \cos^2 \theta + 1 \cos \theta + r}{1} = r \quad (10)$$

$$\Rightarrow r \cos^2 \theta + 1 \cos \theta + r = 1 \Rightarrow r \cos^2 \theta - 1 \cos \theta + 0 = 0 \Rightarrow r (\cos \theta - 1) (\cos \theta - \frac{0}{r}) = 0$$

$$\Rightarrow \boxed{\cos \theta = 1} \quad \therefore \cos \theta = \frac{0}{r} \quad (-1 \leq \cos \theta \leq 1) \text{ في } [0, \pi]$$

$$\text{في النهاية: } BC^2 = AB^2 + AC^2 - 2AB \times AC \cos A \Rightarrow \cancel{r^2} + \cancel{r^2} - 2(r \cos \theta) r$$

$$\Rightarrow r^2 - 2r^2 \cos \theta + r^2 = 2r^2 (1 - \cos \theta) \cos \theta$$

$$\Rightarrow r^2 = \frac{r^2 \cos^2 \theta}{1} - \frac{2r^2 \cos \theta}{1} + \frac{r^2}{1} + \frac{2r^2 \cos \theta}{1} - \frac{2r^2 \cos^2 \theta}{1} = \frac{r^2}{1} = \frac{r^2}{1} \Rightarrow r = \frac{r^2}{r} = \frac{r^2}{r} = r \quad (r = \frac{r^2}{r})$$