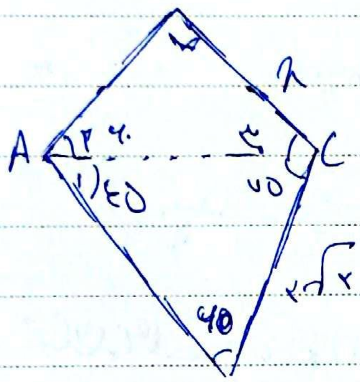
پارسا طوسی — دنیای ریاضی A — تکلیف ۱۳

۱- $B+C=120 \Rightarrow A=60 \Rightarrow \frac{20}{\sin A}=\frac{\frac{20\sqrt{6}}{3}}{\sin B} \Rightarrow \frac{20\sqrt{6}\times\sqrt{3}}{20\times 6}$

$=\frac{\sqrt{2}}{2} \Rightarrow \sin B=\frac{\sqrt{2}}{2} \Rightarrow B=45^\circ \Rightarrow C=75^\circ$



۲- $180-135=45 \Rightarrow A_1=45^\circ$

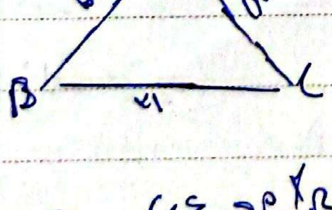
$\frac{2\sqrt{2}}{\sin A_1}=\frac{AC}{\sin 60} \Rightarrow AC=2\sqrt{3}$

$\frac{2\sqrt{3}}{\cancel{\sin 60}}=\frac{x}{\cancel{\sin 30}} \Rightarrow \frac{2\sqrt{3}\times\sqrt{3}}{2}=x=3$

۳- $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k \Rightarrow \frac{(b^2-2)\sin C}{\sin B}=c \Rightarrow \frac{(b^2-2)\sin C}{\sin B}$

$=k\sin C \Rightarrow b^2-2=k\sin B=b$

$\Rightarrow b^2-2-b=0 \Rightarrow b=2$, $b=-1$ غ ق ق $\Rightarrow b=2$



۴- $13^2=21^2+20^2-2(20\times21\times\cos B)$ ⇐ قانون کسینوس

$\Rightarrow \cos B=0.8 \Rightarrow \cos^2 B+\sin^2 B=1$

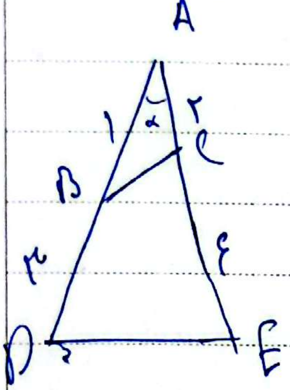
$\Rightarrow 0.64+\sin^2 B=1 \Rightarrow \sin^2 B=0.36 \Rightarrow \sin B=0.6$

۵- $a=\sqrt{16+16+8\sqrt{3}-8-8\sqrt{3}}=\sqrt{24}\Rightarrow 2\sqrt{6}$

$\dfrac{2\sqrt{6}}{\sin A}=\dfrac{4}{\sin C}\Rightarrow \sin C=\dfrac{\sqrt{2}}{2}\Rightarrow C=45^\circ$

$\dfrac{\sqrt{3}}{2}$ $\quad 180-(45+60)=75\Rightarrow B=75^\circ$

۶- $BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos\alpha$



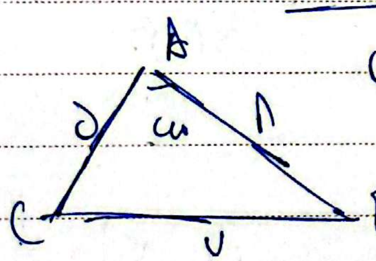
$\Rightarrow 4=4+1-4\cos\alpha\Rightarrow\cos\alpha=\dfrac{1}{4}$

$DE^2=\cancel{AB}\ AD^2+AE^2-2AD\cdot AE\cos\alpha=16+36$

$-12=40\Rightarrow DE=\sqrt{40}=2\sqrt{10}$

(جواب)

۷- الف) $BC=\sqrt{25+64}$ طبق قانون کسینوس



$+(-80\times\dfrac{1}{2})=7\Rightarrow BC=7$

ب)

$S=\dfrac{1}{2}\times5\times8\times\dfrac{\sqrt{3}}{2}=10\sqrt{3}$

۸- طبق قانون کسینوس $\Rightarrow a^2=b^2+c^2-2bc\cos A$

(ادامه صفحه بعد)

$\dfrac{(b+c)^2-a^2}{bc(\cos A+1)}=\dfrac{b^2+2bc+c^2-\cancel{a^2}-b^2-c^2+2bc\cos A}{bc(\cos A+1)}$

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$$= \frac{2(bc + bc\cos A)}{bc(\cos A + 1)}$$ bc تا آخر می بریم ← از صورت و مخرج ساده می شود ← $= 2$

۹- قانون کسینوس ⇐ $a^2 = b^2 + c^2 - 2bc\cos A$

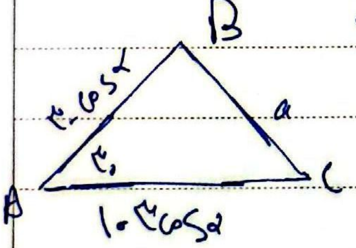
$$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2 \Rightarrow a^3 + b^3 - c^3 = a^3 + a^2 b - a^2 c$$ با سمت راست a^3

$$\Rightarrow b^3 - c^3 = a^2(b - c) \xrightarrow{\text{چاق و لاغر}} (\cancel{b-c})(b^2 + c^2 + bc) = a^2(\cancel{b-c})$$ تا آخر می بریم

$$\Rightarrow b^2 + c^2 + bc = a^2 \Rightarrow \cancel{b^2} + \cancel{c^2} + bc = \cancel{b^2} + \cancel{c^2} - 2bc\cos A \Rightarrow$$

$$bc = -2bc\cos A \Rightarrow \cos A = -\frac{1}{2}$$

۱۰-



$$S = 2 \Rightarrow \frac{(3 - \cos\alpha)(1 + 3\cos\alpha)}{4} = 2$$

$$\Rightarrow 3 + 9\cos\alpha - \cos\alpha + 3\cos^2\alpha = 8$$

$$\Rightarrow 8\cos\alpha + 3\cos^2\alpha = 5$$

$$3\cos^2\alpha + 8\cos\alpha - 5 = 0 \Rightarrow \cos^2\alpha + 8\cos\alpha - 15 = 0 \Rightarrow (\cos\alpha - 3)(\cos\alpha - 5)$$

$$\Rightarrow \cos\alpha = 1 \text{ یا } \cos\alpha = \frac{5}{3}$$ غ ق ق $$\Rightarrow 4 + 16 - \frac{16\sqrt{3}}{2} = 20 - 8\sqrt{3}$$

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