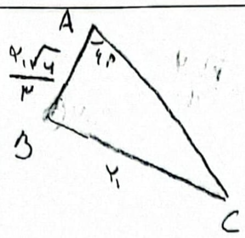
نام و نام خانوادگی ...امیرعلی کیوان گرد....... پاسخنامه تشریحی تکلیف شماره ...۳... کلاس ...دوازدهم تجربی A...

۱

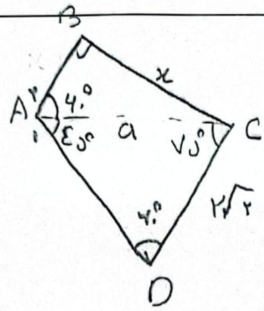


$$\frac{20}{\sin\hat A} = \frac{\frac{20\sqrt6}{3}}{\sin\hat C} \Rightarrow \frac{20}{\frac{\sqrt3}{2}} = \frac{\frac{20\sqrt6}{3}}{\sin\hat C} \Rightarrow \sin\hat C = \frac{\sqrt2}{2} \Rightarrow C = 45^\circ$$

$$\Rightarrow \hat B = 75^\circ$$

$$\hat B + \hat C = 120 \Rightarrow 120 - 45^\circ = 75^\circ$$

۲



$$A_1 = 180 - \hat C - \hat D = 45^\circ$$

$$\frac{2\sqrt2}{\sin\hat A_1} = \frac{a}{\sin\hat D} \Rightarrow \frac{2\sqrt2}{\frac{\sqrt2}{2}} = \frac{a}{\frac{\sqrt3}{2}} \Rightarrow a = 2\sqrt3$$

$$\sin A_2 = \frac{x}{a} \Rightarrow \frac{\sqrt3}{2} = \frac{x}{2\sqrt3} \Rightarrow x = 3$$

۳

$$\frac{a}{\sin\hat A} = \frac{b}{\sin\hat B} = \frac{c}{\sin\hat C} \Rightarrow c\sin\hat B = b\sin\hat C \quad (1)$$

$$\boxed{(b^2-2)}\sin\hat C = \sin\hat B\, c \quad (2)$$

$$\Rightarrow b^2-2 = b \Rightarrow b^2-b-2=0$$

$$(b-2)(b+1)=0$$

$$\boxed{b=2} \checkmark \qquad b=-1 \times$$

۴

$$P = \frac{20+21+13}{2} = 27 \qquad S_\Delta = \sqrt{P(P-a)(P-b)(P-c)}$$

نصف محیط

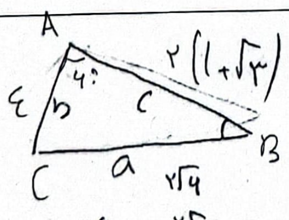
طبق قضیه هرون می‌دانیم

$$\Rightarrow S_\Delta = \sqrt{27(7)(6)(14)}$$

$$S_\Delta = \sqrt{3^4\times7^2\times2^2} = 7\times2\times9 = 126$$

$$S_\Delta = AB\times BC\times\sin\hat B\times\frac12 \Rightarrow \cancel{20\times21}\times\sin\hat B = 2\times7\times2\times9 \Rightarrow \sin\hat B = \frac{3}{5}$$

۵

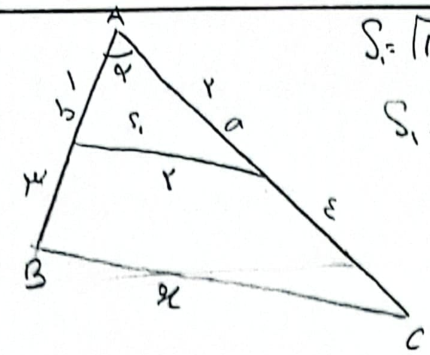


$$a^2 = b^2+c^2-\cancel{2}bc\cos\hat A = b^2+c^2-bc$$

$$16+4+12+8\sqrt3-8-8\sqrt3 = 24 \Rightarrow \boxed{a=2\sqrt6}$$

$$\frac{4}{\sin\hat B} = \frac{2\sqrt6}{\underbrace{\sin\hat A}_{\frac{\sqrt3}{2}}} \Rightarrow \frac{4}{\sin\hat B} = 4\sqrt2 \Rightarrow \sin\hat B = \frac{\sqrt2}{2} \Rightarrow \boxed{\hat B = 45^\circ}$$

$$\hat C = 180-(60+45) = \boxed{75^\circ}$$

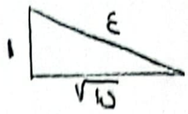


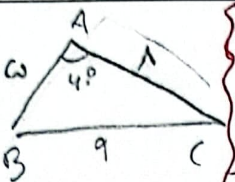
$S_1=\sqrt{p(p-a)(p-b)(p-c)} = \sqrt{\frac{5}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{3}{2}} = \frac{\sqrt{15}}{4}$

$S_1 = \frac{ab\sin\alpha}{2} \Rightarrow 1\times2\times\frac{1}{2}\times\sin\alpha = \frac{\sqrt{15}}{4} \Rightarrow \sin\alpha=\frac{\sqrt{15}}{4} \Rightarrow \cos\alpha=\frac{1}{4}$

$x^2 = AB^2+AC^2-2AB\,AC\cos\alpha$ ۶

$16+36-12 \Rightarrow x^2=40 \Rightarrow x=2\sqrt{10}$

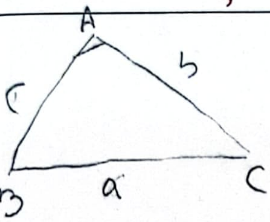




الف) $BC^2=AB^2+AC^2-2AB\,AC\cos60^\circ = 25+64-10=81$

$\Rightarrow BC=9$

ب) $S_\Delta = AB\times AC\times\sin\hat A\times\frac{1}{2} = 5\times8\times\frac{1}{2}\times\frac{\sqrt3}{2}=10\sqrt3$ ۷



$a^2=b^2+c^2-2bc\cos\hat A$; $\frac{b^2+c^2+2bc-b^2-c^2+2bc\cos\hat A}{bc(2\cos\hat A+1)}$ ①

$(b+c)^2=b^2+c^2+2bc$ ①

$=\frac{bc(2+2\cos\hat A)}{bc(1\cos\hat A+1)} = \frac{2+2\cos\hat A}{1+1\cos\hat A} = \boxed{2}$ ۸

$\frac{(b-c)\overbrace{(b^2+c^2+bc)}^{a^2}+a^3}{(b+a-c)} = \frac{a^2\cancel{(b+a-c)}}{\cancel{(b+a-c)}} = a^2$ ۹

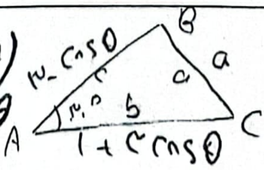
۲ $\Leftarrow$ $\Leftrightarrow$

اثبات بازگشتی انجام می‌دهیم

از این می‌توان نتیجه گرفت که

$\cos\hat A=-\frac{1}{2}$

$S^{\Delta}_{ABC}=\frac{1}{2}AB\times AC\times\sin\hat A \Rightarrow 2 = \frac{1}{2}\times\frac{1}{2}\times\frac{(3-\cos\theta)(1+4\cos\theta)}{3+9\cos\theta-\cos\theta-4\cos^2\theta}$



$\Rightarrow -4\cos^2\theta+8\cos\theta+3=8 \Rightarrow 4\cos^2\theta-8\cos\theta+5=0 \Rightarrow \cos\theta=1$ ۱۰

$a^2=AB^2+AC^2-2AB\,AC\cos\hat A \Rightarrow a^2=4+16-2\times2\times4\times\frac{\sqrt3}{2} \Rightarrow a^2=20-8\sqrt3$