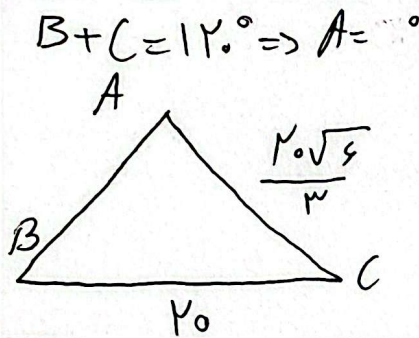




$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{20}{\sin 60^\circ} = \frac{10\sqrt{3}}{\sin B}$$

$$\Rightarrow \sin B = \frac{\sqrt{12}}{2} = \frac{3\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \Rightarrow B=45^\circ$$

$C=75^\circ$

$A=180-(90+75)=15^\circ$

$$\frac{CD}{\sin A} = \frac{AC}{\sin 90^\circ} \Rightarrow \frac{CD}{\sin 15^\circ} = \frac{AC}{1}$$

$$\Rightarrow \frac{CD}{\sin 15^\circ} = \frac{AC}{1} \Rightarrow \frac{CD}{\frac{1}{2}} = AC \Rightarrow AC=2CD$$

$$\frac{x}{\sin 90^\circ} = \frac{2\sqrt{2}}{\sin 90^\circ} \Rightarrow \frac{x}{1} = \frac{2\sqrt{2}}{1} \Rightarrow x=2\sqrt{2}$$

$x = \frac{\sqrt{2}}{2} \times 2\sqrt{2} = 2$

$$\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{a}{\sin A} = k \Rightarrow c = k \sin C \Rightarrow (b \neq 2) \frac{\sin C}{\sin B} = k \sin C$$

$$\Rightarrow \frac{b^2 - 2}{\sin B} = k \Rightarrow b^2 - 2 = k \sin B \Rightarrow b^2 - b - 2 = 0 \Rightarrow (b-2)(b+1) = 0$$

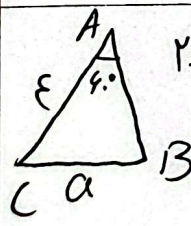
$\Rightarrow b=2$ ✓
 $b=-1$ ✗

$$AC^2 = BC^2 + AB^2 - 2BC \cdot AB \cdot \cos B$$

$$149 = 49 + 4 - 2(7 \cdot 2 \cdot \cos B) \Rightarrow -67 = -14 \cos B$$

$$\Rightarrow \cos B = \frac{67}{14} \rightarrow \frac{67}{14} + \sin^2 B = 1 \Rightarrow \sin^2 B = \frac{34}{14} \Rightarrow \sin B = \frac{\sqrt{34}}{2}$$

Case 1) $B < 90^\circ$ ✓



الف) $a^2 = b^2 + c^2 - 2bc \cos A = 14 + 4 + 14 - 2(7)(2)\cos 60^\circ$

$$\Rightarrow 14 + 14 + 14 - 14 = 28 \Rightarrow a^2 = 28 \Rightarrow a = \sqrt{28} = 2\sqrt{7}$$

ب) $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2\sqrt{7}}{\sin 60^\circ} = \frac{2}{\sin B} \Rightarrow \sin B = \frac{1}{\sqrt{7}}$

$$\Rightarrow \sin B = \frac{\sqrt{7}}{7} \Rightarrow B=90^\circ$$

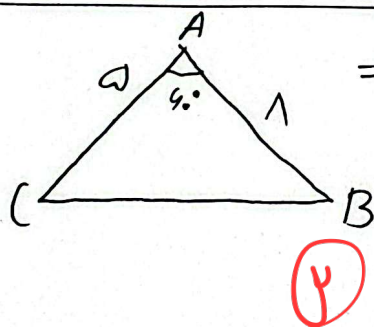
$C=90^\circ$

$$BC^2 = AC^2 + AB^2 - 2AC \times AB \times \cos \alpha \Rightarrow E = E + 1 - 2(2 \times 1 \times \cos \alpha)$$

$$\Rightarrow -1 = -2 \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2} \quad \checkmark$$

$$DE^2 = AE^2 + AD^2 - 2AE \times AD \times \cos \alpha = 4 + 1 - 2(2 \times 1 \times \frac{1}{2})$$

$$\Rightarrow DE^2 = 2 - 1 = 1 \Rightarrow DE = \sqrt{1} = 1 \quad \checkmark$$



$$\Rightarrow BC^2 = (1)^2 + (2)^2 - 2 \times 1 \times 2 \times \cos 60^\circ$$

$$\Rightarrow BC^2 = 1 + 4 - 2 = 3 \Rightarrow BC = \sqrt{3} \quad \checkmark$$

$$S = \frac{1}{2} \times 1 \times 2 \times \sin 60^\circ = \frac{1 \times \sqrt{3}}{2} = \frac{\sqrt{3}}{2} \quad \checkmark$$

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} \Rightarrow (b+c)^2 - a^2 = b^2 + c^2 + 2bc - a^2$$

$$= (b^2 + c^2 + 2bc) - (b^2 + c^2 - 2bc \cos A) = 2bc(1 + \cos A)$$

$$\Rightarrow \frac{2bc(1 + \cos A)}{bc(\cos A + 1)} = 2 \quad \checkmark$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a \Rightarrow a^2 + b^2 - c^2 = a(a+b-c) \Rightarrow a^2 + b^2 - c^2 = a^2 + ab - ac$$

$$\Rightarrow b^2 - c^2 = ab - ac = a(b-c) \Rightarrow (b-c)(b+c) = a(b-c)$$

$$\Rightarrow (b-c)(b+c-a) = 0 \quad \frac{b \neq c}{b-c \neq 0} \Rightarrow b+c-a = 0 \Rightarrow b+c = a$$

$$\Rightarrow b+c = a \Rightarrow b+c = a \Rightarrow A = 120^\circ \quad \checkmark$$

$$S = 2 \Rightarrow \frac{1}{2} \times (1 + 2 \cos \theta) \times (2 \cos \theta) \times \frac{1}{2} = 2 \Rightarrow -2 \cos^2 \theta + 2 \cos \theta + 1 = 2$$

$$\Rightarrow -2 \cos^2 \theta + 2 \cos \theta - 1 = 0 \Rightarrow 2 \cos^2 \theta - 2 \cos \theta + 1 = 0 \Rightarrow (2 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad \checkmark$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow a^2 = (1)^2 + (2)^2 - 2(1 \times 2 \times \cos 60^\circ)$$

$$\Rightarrow a^2 = 1 + 4 - 2 = 3 \Rightarrow a = \sqrt{3} \quad \checkmark$$