محمد عماد عموریا — دوازدهم ریاضی

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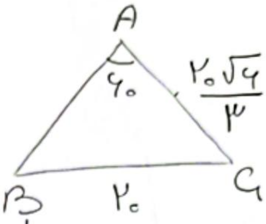

$\hookrightarrow B + C = 120$

$\frac{20}{\sin 60} = \frac{\frac{20\sqrt{6}}{3}}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$

($\sin 60 = \frac{\sqrt{3}}{2}$)

$\Rightarrow 45 + C = 120 \rightarrow \boxed{C = 75^\circ}$

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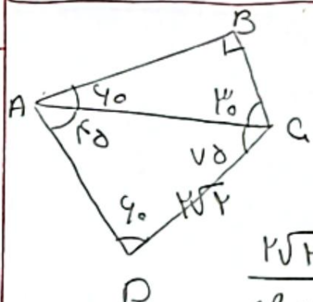

$A + B + C + D = 360$ (90, 105, 90)

$\hookrightarrow A = 105$

$\frac{2\sqrt{2}}{\sin 45} = \frac{AC}{\sin 60} \rightarrow AC = 2\sqrt{3}$ ($\sin 45 = \frac{\sqrt{2}}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$)

$\sin 60 = \frac{BC}{AC} = \frac{\sqrt{3}}{2} \rightarrow \boxed{BC = 3}$ ($AC = 2\sqrt{3}$)

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$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = m \rightarrow b = m \sin B$

$\hookrightarrow c = m \sin C$

$\frac{(b^2 - 2)\sin C}{\sin B} = c \Rightarrow b^2 - 2 = m \sin B = b$ ($c = m \sin C$)

$b^2 - b - 2 = 0$ → $\boxed{b = 2}$

$\hookrightarrow b = -1 \rightarrow$ غ ق ق

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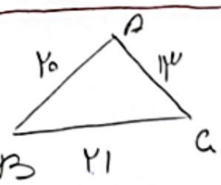

$AC^2 = 21^2 + 20^2 - (2 \times 21 \times 20 \times \cos B)$ ($AC^2 = 13^2$)

$\hookrightarrow \cos B = 0.8$

$\cos^2\alpha + \sin^2\alpha = 1 \rightarrow \frac{16}{25} + \sin^2 B = 1$

$\Rightarrow \sin^2 B = 9/25 \rightarrow \sin B = \frac{3}{5} = \boxed{0.6}$

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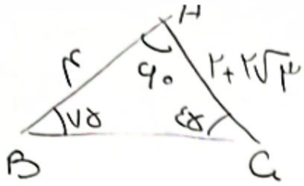

$BC = \sqrt{AB^2 + AC^2 - 2AB\,AC\cos 60}$

$\Rightarrow \sqrt{16 + 16 + 8\sqrt{3} - 8 - 8\sqrt{3}} \Rightarrow BC = 2\sqrt{6}$

$\frac{2\sqrt{6}}{\sin 60} = \frac{4}{\sin C} \Rightarrow \sin C = \sqrt{2}/2 \rightarrow C = 45^\circ$ ($\sin 60 = \frac{\sqrt{3}}{2}$)

$\Rightarrow 45 + 60 + B = 180 \rightarrow \boxed{B = 75^\circ}$

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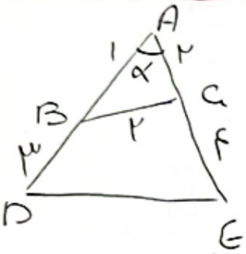

$BC^2 = AB^2 + AC^2 - 2AB\,AC\cos\alpha$

$\Rightarrow 4 = 4 + 1 - 4\cos\alpha$

$\cos\alpha = 1/4$

$DE^2 = AD^2 + AE^2 - 2AD\,AE\cos\alpha$

$\Rightarrow 16 + 36 - (48 \times \frac{1}{4}) = 40$

$\Rightarrow DE = \sqrt{40} \rightarrow \boxed{2\sqrt{10}}$

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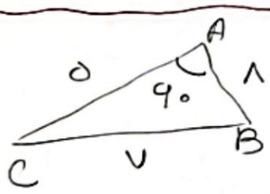

$BC^2 = AB^2 + AC^2 - 2AB\,AC\cos 60$

$\Rightarrow 25 + 64 - (80 \times \frac{1}{2}) = 49$

$\Rightarrow \boxed{BC = 7}$

$S = \frac{1}{2} \times 8 \times 5 \times \sin 60 = \boxed{10\sqrt{3}}$ ($\sin 60 = \frac{\sqrt{3}}{2}$)

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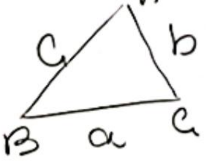

$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)}$ $\quad a^2 = c^2 + b^2 - 2bc\cos A$

$\frac{b^2 + c^2 + 2bc - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - (c^2 + b^2 - 2bc\cos A)}{bc(\cos A + 1)}$

$\frac{2bc + 2bc\cos A}{bc(\cos A + 1)} = \boxed{2}$

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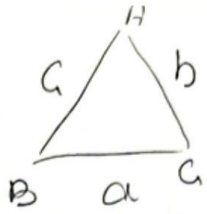

$$\frac{a^2+b^2-c^2}{a+b-c} = a^2$$

$$\rightarrow a^2+b^2-c^2 = a^2+a^2b-a^2c$$

$$(b-c)(b^2+c^2+bc) = a^2(b-c)$$

$$\Rightarrow b^2+c^2+bc = a^2$$

$$a^2 = b^2+c^2-2bc\cos A$$

$$\Rightarrow \cos\hat{A} = \boxed{-1/2}$$

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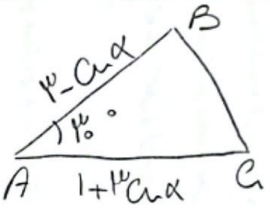

$$S = \frac{1}{2}\times AB\times AC\times \sin 120^\circ = 2$$

$$\Rightarrow (2-\cos\alpha)(1+2\cos\alpha) = 1$$

$$2\cos^2\alpha - \cos\alpha + 2 = 0 \rightarrow \cos^2\alpha - \cos\alpha + 1 = 0$$

$$(\cos\alpha - 2)(\cos\alpha - \tfrac{1}{2}) = 0$$

$$\cos\alpha = \frac{2}{2} = 1 \qquad \cos\alpha = 1/2 \text{ ق ق ی}$$

$$\Rightarrow BC^2 = 4^2 + 14 - 2(4)(2)\left(\frac{\sqrt{2}}{2}\right) = \boxed{20 - 8\sqrt{2}}$$