

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{20}{\frac{\sqrt{2}}{2}} = \frac{\frac{20\sqrt{2}}{2}}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2}$$

$$\Rightarrow B = 45^\circ$$

$$C = 115^\circ$$

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$$\frac{DC}{\sin A} = \frac{AC}{\sin B} \Rightarrow \frac{2\sqrt{2}}{\sin 45^\circ} = \frac{AC}{\sin 45^\circ} \Rightarrow AC = 2\sqrt{2}$$

$$\frac{AC}{\sin B} = \frac{BC}{\sin A} \Rightarrow \frac{2\sqrt{2}}{\sin 90^\circ} = \frac{n}{\sin 45^\circ} \Rightarrow n = 2$$

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$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b \times \frac{\sin C}{\sin B} = c$$

$$(b^2 - 4) \times \frac{\sin C}{\sin B} = c \Rightarrow b = \begin{cases} 2 \\ -1 \end{cases} \checkmark$$

غیر قابل قبول -۱

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$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$13^2 = 20^2 + 21^2 - 2 \times 20 \times 21 \times \cos B \Rightarrow \cos B = 0/1$$

$$\cos^2 B + \sin^2 B = 1 \Rightarrow \sin B = \begin{cases} -0.19 \\ +0.19 \end{cases}$$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 14 + (2 + 2\sqrt{2})^2 - 2 \times 2 \times (2 + 2\sqrt{2}) \times \frac{1}{2} \Rightarrow a = 2\sqrt{2}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{2}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$$

$$\Rightarrow C = 115^\circ$$

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$$\text{مثلاً كوساينك : } r^2 = r^2 + 1^2 - 2 \times r \times 1 \times \cos \alpha \Rightarrow \cos \alpha = \frac{1}{r}$$

$$\text{مثلاً بزرگ : } DE^2 = 9^2 + 5^2 - 2 \times 9 \times 5 \times \underbrace{\cos \alpha}_{\frac{1}{r}}$$

$$\Rightarrow DE = 2\sqrt{10}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 1^2 + 2^2 - 2 \times 1 \times 2 \times \frac{1}{r} \Rightarrow a = \sqrt{r}$$

$$S = \frac{1}{r} b \times c \times \sin A \quad S = \frac{1}{r} \times 1 \times 2 \times \frac{\sqrt{r}}{r} = 10\sqrt{r}$$

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc + a^2}{bc(\cos A + 1)} = \frac{\overbrace{b^2 + c^2 - 2bc \cos A}^{a^2} + 2bc \cos A + 2bc}{bc(\cos A + 1)}$$

$$= \frac{2bc(\cos A + 1)}{bc(\cos A + 1)} = 2$$

$$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \Rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c$$

$$\Rightarrow b^2 - c^2 = a^2(b - c)$$

$$\Rightarrow (b - c)(b + c) = a^2(b - c)$$

$$b^2 + c^2 + bc = a^2 \Rightarrow b^2 + c^2 + bc = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow \cos A = -\frac{1}{r}$$

$$S = r = \frac{1}{r} \sin 90^\circ \times (r - \cos \theta) \times (1 + r \cos \theta) \Rightarrow \cos \theta = 1$$

$$\Rightarrow AB = r$$

$$\Rightarrow AC = r$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = r^2 + r^2 - 2 \times r \times r \times \frac{\sqrt{r}}{r} = 20 - 1\sqrt{r}$$