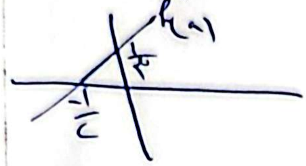
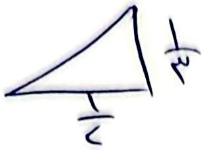
نام و نام خانوادگی ...آرین رحیمه پولادگر... پاسخنامه تشریحی تکلیف شماره .... کلاس ...دوازدهم ریاضی...

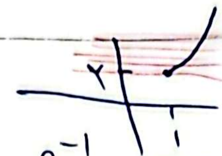
$f(x) = \frac{3}{2}x - \frac{1}{2} \longrightarrow f^{-1}(x) = \frac{2}{3}\left(x + \frac{1}{2}\right) = \frac{2}{3}x + \frac{1}{3}$





$S = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

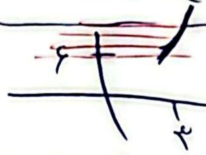
الف) $f(x) = x^2 - 2x + 3 \quad x \in [1, +\infty)$



یک به یک است

$f(x) = x^2 - 2x + 1 + 2 = (x-1)^2 + 2 \Longrightarrow f^{-1}(x) = \sqrt{x-2} + 1$

$D_{f^{-1}} = R_f = [2, +\infty)$



یک به یک است

ب) $y = x^2 - 2x + 3 \quad x \in [3, +\infty)$

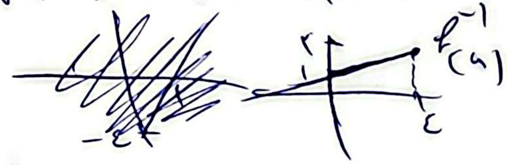
$f(x) = x^2 - 2x + 3 \to x^2 - 2x + 1 + 2 = (x-1)^2 + 2 \to f^{-1}(x) = \sqrt{x-2} + 1 \quad D_{f^{-1}} = R_f = [6, +\infty)$

$f(x) = 2x - \sqrt{16 - 4x(4-x)} = 2x - \sqrt{16 - 16x + 4x^2}$

$f(x) = 2x - 2\sqrt{x^2 - 4x + 4} = 2x - 2\sqrt{(x-2)^2} = 2x - 2|x-2|$

$2x - |2x - 4|$

$f(x) = \begin{cases} 4 & x > 2 \\ \cancel{2x + 2x - 4} \;\; 4x - 4 & x \leq 2 \end{cases}$



$f^{-1}(x) = \frac{x+4}{4} = \frac{x}{4} + 1 \quad x \leq 4$

$S_{\text{ذوزنقه}} = \frac{1}{2}(1+2)\,4 = 6 \longrightarrow$ جواب

$y_1 = y_2 \Longrightarrow x_1 = x_2 \qquad \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \Longrightarrow \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5}$

$(x_1 x_2)^2 - 5x_1^2 = (x_1 x_2)^2 - 5x_2^2 \Longrightarrow x_1 = \pm x_2$ یک به یک است

$x_1^2 - 5 \geq 0 \Longrightarrow x^2 \geq 5 \to \boxed{|x| \geq \sqrt{5}} \quad x_1 = -x_2 \; ✗$

در واقع x_1 و x_2 نمی‌توانند مختلف العلامت داشته باشند پس $x_2 = -x_1$ غلط است

$y = \frac{x}{\sqrt{x^2 - 5}} \Longrightarrow y^2 = \frac{x^2}{x^2 - 5} \Longrightarrow x^2y^2 - 5y^2 = x^2 \longrightarrow x^2(y^2 - 1) = 5y^2$

$x = \pm\frac{\sqrt{5}\,y}{\sqrt{y^2 - 1}} \xrightarrow{x,y} y = \frac{\sqrt{5}\,x}{\sqrt{x^2 - 1}}$

هم علامت

$3k + 8 \leq k - 4$

$2k \leq -12 \Longrightarrow \boxed{k \leq -6}$

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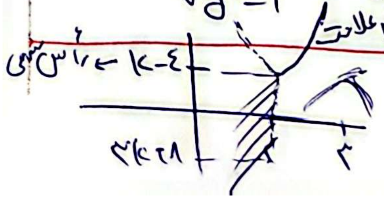
در این حالت تابع یک به یک است



$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \\ -x^2 + 6x + 3k & x < 2 \end{cases} \quad \to -\frac{b}{2a} = \frac{-6}{-2} = 3$

(۶)

$f(x) = 3x + |x| \Rightarrow f(x) = \begin{cases} 4x & x \geq 0 \\ 2x & x < 0 \end{cases} \longrightarrow f^{-1}(x) = \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{2} & x < 0 \end{cases}$

دو نقطه کم داریم

$f(x) \to 1 \to 4 \quad f^{-1}(x)\ 4 \to 1 \quad f(x) \to -1 \to -2 \quad f^{-1}(x) \to -2 \to -1$

$4a + 4b = 1 \qquad 4a + 4b = 1$

$-2a + 2b = -1 \xrightarrow{\times 2} -4a + 4b = -2 \to 8b = -1 \to b = -\frac{1}{8} \to a = \frac{3}{8}$

$ab = \frac{3}{8} \times -\frac{1}{8} = -\frac{3}{64}$

(۷)

$f(x) = \frac{3x+4}{x+m} \xrightarrow{12 \to -10} \frac{10}{2+m} = -10 \to 2+m = -1 \to \boxed{m = -3}$

$f(x) = \frac{3x+4}{x-3} \qquad y = h \circ f^{-1}(x) + f^{-1}(x) = x + f^{-1}(x) = x \Rightarrow f^{-1}(x) = 0$

$a = ? \quad f(0) = a \to \boxed{\frac{-4}{3} = a} \to$ جواب

(۸)

$f(3-2x) + g(\frac{x}{2}) = 2 \qquad f(3-2x) = a \qquad f^{-1}(a) = 3 - 2x$

$g(\frac{x}{2}) = 2 - a \to g^{-1}(2-a) = \frac{x}{2} \to x = 2g^{-1}(2-a)$ (1) $\qquad x = \frac{f^{-1}(a) - 3}{-2}$ (2)

$\frac{f^{-1}(a) - 3}{-2} = 2g^{-1}(2-a) \Rightarrow f^{-1}(x) + 4g^{-1}(2-x) = 3 \xrightarrow{x=-2} f^{-1}(-2) + 4g^{-1}(4) = \boxed{3}$

جواب

(۹)

قرینه نسبت به مبدأ

$-\frac{-5x-13}{-(-x)+1} = \frac{5x+13}{x+1} \to \frac{5(x-2)+13}{(x-2)+1} - 3 = \frac{5x+3}{x-1} - 3$

$\frac{5x+3-3x+3}{x-1} = \frac{2x+6}{x-1} = h(x)$ برای پیدا کردن نقاط برخورد دو نمودار $h(x)$ را مساوی x قرار دادیم $\qquad \frac{2x+6}{x-1} = x \Rightarrow x^2 - x = 2x + 6$

$x^2 - 3x - 6 = 0$ مجموع نقاط برخورد $\boxed{3 = \frac{-b}{a}} \longrightarrow$ جواب

(۱۰)

$f(x) = x + 4[x] \qquad f(x) - g(x) = ?$

$x = y + 4[y]$ (1) $\to [x] = [y + 4[y]] = [x] = 5[y] \to [x] = \frac{5}{4}(x-y)$

$\to [y] = \frac{x-y}{4}$

$\frac{5}{4}y = -[x] + \frac{5}{4}x \longrightarrow y = -\frac{4}{5}[x] + x = g(x)$

$f(x) - g(x) = \cancel{x} + 4[x] - \cancel{x} + \frac{4}{5}[x] = \frac{24}{5}[x] = 4/8[x]$