

$$y = \frac{x}{2} - \frac{1}{y} = f(x) \Rightarrow f^{-1}(x) \Rightarrow x = \frac{y}{2} - \frac{1}{y} \Rightarrow x + \frac{1}{y} = \frac{y}{2} \Rightarrow y = \frac{x}{2} + \frac{1}{y} = f^{-1}(x)$$

$$\Rightarrow x=0 \Rightarrow y=\frac{1}{y}$$

$$y=0 \Rightarrow x=-\frac{1}{y} \Rightarrow \Rightarrow S = \frac{\frac{1}{y} \times \frac{1}{y}}{2} = \frac{1}{12}$$

$$x_5 = -\frac{y}{x} = \frac{1}{y} = 1 \Rightarrow$$

$$f(x) = x^2 + x + 1 = (x-1)^2 + 2 \quad f^{-1}(x) = \sqrt{x-2} + 1$$

$$\Rightarrow f^{-1} \Rightarrow x = (y-1)^2 + 2 \Rightarrow f^{-1}(x) = \sqrt{x-2} + 1$$

$$Df^{-1} = [1, +\infty)$$

$$Df^{-1} = Rf = (x-1)^2 + 2 = [2, +\infty)$$

$$f(x) = 2x - \sqrt{x^2 - 4x + 4} = 2x - 2\sqrt{(x-2)^2} = 2x - 2|x-2|$$

$$f(x) = \begin{cases} 4x-4 & x \leq 2 \\ 4 & x \geq 2 \end{cases} \Rightarrow (-\infty, 2] \cup [4, +\infty) \Rightarrow Df^{-1} = Rf = [-\infty, 4]$$

$$f^{-1} \Rightarrow x = f(y) - 4 \Rightarrow \frac{x+4}{4} = y \Rightarrow f^{-1}(x) = \frac{x+4}{4}$$

$$S_{\text{مجموع}} = \frac{(1+2) \times 4}{2} = 6$$

$$\frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \Rightarrow x_1 \sqrt{x_2^2 - 5} = x_2 \sqrt{x_1^2 - 5} \Rightarrow x_1^2 (x_2^2 - 5) = x_2^2 (x_1^2 - 5)$$

$$\Rightarrow x_1^2 x_2^2 - 5x_1^2 = x_1^2 x_2^2 - 5x_2^2 \Rightarrow -5x_1^2 = -5x_2^2 \Rightarrow x_1^2 = x_2^2$$

$$x_1 = x_2 \vee x_1 = -x_2 \Rightarrow x_1 = x_2 \rightarrow \text{نقض} \Rightarrow x > \sqrt{5} \text{ و } x_1 = x_2 \Rightarrow x < -\sqrt{5}$$

۴) بخش معکوس تابع

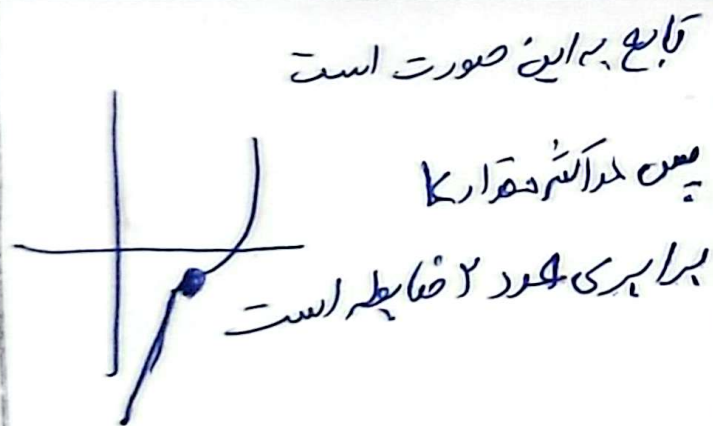
$$y\sqrt{x^2 - 5} = x \Rightarrow y^2(x^2 - 5) = x^2 \Rightarrow y^2 x^2 - 5y^2 = x^2$$

$$y^2 x^2 - x^2 = 5y^2 \Rightarrow x^2 = \frac{5y^2}{y^2 - 1} \Rightarrow x = \pm \sqrt{\frac{5y^2}{y^2 - 1}}$$

$$\Rightarrow y = \pm \frac{\sqrt{5}x}{x^2 - 1} \Rightarrow x > \sqrt{5} \Rightarrow y > 0$$

$$x < -\sqrt{5} \Rightarrow y < 0$$

$$\Rightarrow y = \frac{\sqrt{5}x}{x^2 - 1} = f^{-1}(x)$$

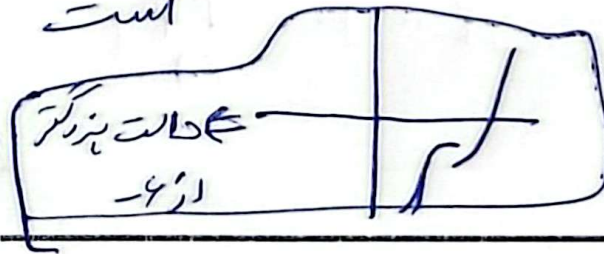


$$x=2 \Rightarrow 4-1+k \Rightarrow 4+12+3k$$

$$3k = -12 \Rightarrow k = -4$$

$$\Rightarrow \text{محدوده مقدار } k = -4 \Rightarrow k \in (-\infty, -4]$$

است



$$f(x+|x|) \Rightarrow f(x) \Rightarrow \begin{cases} f(x) = x & x \geq 0 \\ f(x) = -x & x \leq 0 \end{cases} \Rightarrow f^{-1}(x) = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

$$\Rightarrow f(x) = ax + b|x| \Rightarrow \begin{aligned} x \geq 0 &\Rightarrow a + b = \frac{1}{x} \\ x \leq 0 &\Rightarrow a - b = \frac{1}{x} \end{aligned} \Rightarrow \begin{aligned} a + b &= \frac{1}{x} \\ a - b &= \frac{1}{x} \end{aligned} \Rightarrow \begin{aligned} 2a &= \frac{2}{x} \Rightarrow a = \frac{1}{x} \\ 2b &= 0 \Rightarrow b = 0 \end{aligned}$$

$$a \wedge b = \frac{-3}{19}$$

$$x \Rightarrow y \Rightarrow \frac{1}{x+m} = -1 \Rightarrow m = -3 \Rightarrow f(x) = \frac{3x+1}{x-3} \Rightarrow a+d = 0$$

$$\Rightarrow f \circ f^{-1}(x) = x \Rightarrow y = x + \frac{3x+1}{x-3} \Rightarrow x + \frac{3x+1}{x-3} = x$$

$$\frac{3x+1}{x-3} = 0 \Rightarrow 3x+1 = 0 \Rightarrow x = -\frac{1}{3}$$

$$\begin{aligned} f(g^{-1}(x)) &= 1 - g(x) \Rightarrow g^{-1}(f) = a \Rightarrow g(a) = f \Rightarrow \frac{x}{y} = a \Rightarrow x = ay \\ \Rightarrow f(1 - f(a)) &= 1 - g(a) \xrightarrow{g(a)=f} f(1 - f(a)) = -1 \Rightarrow f^{-1}(-1) = 1 - f(a) \\ f(g^{-1}(f)) + f^{-1}(1 - f) &= fa + 1 - fa = 1 \end{aligned}$$

$$\textcircled{1} \frac{-5x-13}{x+1} \rightarrow \textcircled{2} \frac{5x+13}{x+1} \Rightarrow \textcircled{3} \frac{5x+13}{x+1} \Rightarrow \textcircled{4} \frac{5x+13}{x+1}$$

$$\Rightarrow \frac{5(x-2x+13)}{x-1+1} - 13 = \frac{5x+13}{x-1} - 13 = \frac{1x+4}{x-1} = f(x) \Rightarrow f^{-1}(x) = \frac{x+4}{x-1}$$

$$\Rightarrow \frac{1x+4}{x-1} = \frac{x+4}{x-1} \Rightarrow (1x+4)(x-1) = (x+4)(x-1) \Rightarrow x^2 - 3x - 4 = 0$$

$$|x_1 + x_2| = \frac{-b}{a} = \boxed{3}$$

$$y = x + f[x] \Rightarrow [y = x + f[x]] \Rightarrow [y] = \omega[x] \Rightarrow [x] = \frac{[y]}{\omega} \textcircled{1}$$

$$\textcircled{1} y = x + f[x] \Rightarrow f[x] = y - x \Rightarrow \textcircled{2} f[x] = y - x$$

$$\textcircled{1} \textcircled{2} \Rightarrow \frac{f[y]}{\omega} = y - x \Rightarrow x = y - \frac{f[y]}{\omega} \Rightarrow f^{-1}(x) = g(x) = x - \frac{f[y]}{\omega}$$

$$(f-g)(x) = x + f[x] - \omega + \frac{f[x]}{\omega} = \frac{f[x]}{\omega} = f_1[x]$$