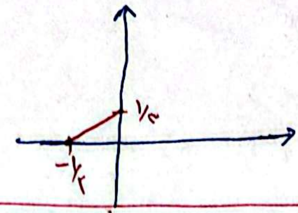
پارسا انوری تکلیف ۱

۱۹٫۷۵

-۱ (۲)

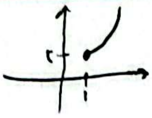
$f_{(x)} = \frac{3x-1}{2} \Rightarrow \frac{2y+1}{3} = x \Rightarrow f^{-1}_{(x)} = \frac{2x+1}{3}$



$S = \frac{1}{2}(\frac{1}{3} \times \frac{1}{2}) = \frac{1}{12}$ ✓

-۲ (۲)

الف) رأس سهمی $= (1,2)$

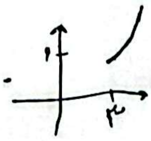


یک به یک است

$f_{(x)} = y = (x-1)^2 + 2 \Rightarrow \sqrt{y-2} + 1 = x \Rightarrow f^{-1}_{(x)} = \sqrt{x-2} + 1$ ✓

$\sqrt{x-2} + 1 \geq 1 \Longrightarrow D_{f^{-1}} = [2, +\infty)$

ب) رأس سهمی $= (1,2)$ ، $3 \geq x_S$



یک به یک است

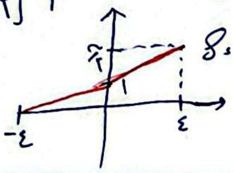
$\Rightarrow f^{-1}_{(x)} = \sqrt{x-2} + 1 \geq 3 \Rightarrow D_{f^{-1}_{(x)}} = [6, +\infty)$ ✓

-۳ (۲)

$f_{(x)} = 3x - \sqrt{16 - 16x + 4x^2} = 3x - 2\sqrt{(x-2)^2} = 3x - 2|x-2|$ $\longrightarrow x \geq 2 \to y = 4$ ✗ یک به یک نیست

$\searrow x < 2 \to y = 4x - 4$ ✓ است

$f_{(x)} = 4x - 4$ ، $D_{f_{(x)}} = (-\infty, 2]$ $\Rightarrow f^{-1}_{(x)} = \frac{x+4}{4} = \frac{x}{4} + 1$ ، $D_{f_{(x)}} = R_{f^{-1}_{(x)}} \Longrightarrow \frac{x}{4} + 1 \leq 2 \Rightarrow x \leq 4 \Rightarrow f^{-1}_{(x)} = \frac{x}{4} + 1$ ، $D_{f^{-1}_{(x)}} = (-\infty, 4]$



$S = \frac{4}{2} = 2$ مساحت با جهت مثبت محورها

!! این تابع برخلاف خواسته سوال، جهت مثبت محور x ها را قطع نمی‌کند.

مساحت محدود در جهت مثبت محور x ها: $\frac{1}{2}(1+\frac{3}{2})(4) = 6$ ✓ ۵

-۴ (۲)

$y_2 = y_1 \to \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \xrightarrow{\sqrt{x_1^2-5} > 0 ,\ \sqrt{x_2^2-5} > 0} x_1, x_2$ هم علامت اند (I) $\Rightarrow \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5} \Rightarrow$

$x_1^2 x_2^2 - 5x_1^2 = x_2^2 x_1^2 - 5x_2^2 \Rightarrow x_1^2 = x_2^2 \xrightarrow{I} x_1 = x_2 \Rightarrow$ تابع یک به یک است

$y^2 = \frac{x^2}{x^2-5} \Rightarrow y^2 x^2 - 5y^2 = x^2 \Rightarrow x^2(1-y^2) = -5y^2$

$x^2 = \frac{-5y^2}{1-y^2} \Rightarrow \frac{5y^2}{y^2-1} \Rightarrow x = \frac{y\sqrt{5}}{\sqrt{y^2-1}} \Rightarrow f^{-1}_{(x)} = \frac{\sqrt{5}x}{\sqrt{x^2-1}}$ ✓

-۵ (۲)

$4 - 8 + k \geq -4 + 12 + 2k \Rightarrow -12 \geq 2k \Rightarrow k \leq -6$ ✓

-۶ (۲)

$3x + |x| \begin{cases} 4x & x \geq 0 \\ 2x & x < 0 \end{cases} \Rightarrow f^{-1}_{(x)} = \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{2} & x < 0 \end{cases} \Rightarrow \frac{\frac{x}{4} + \frac{x}{2}}{2} = \frac{3x}{8} \Rightarrow f^{-1}_{(x)} = \frac{3}{8}x - \frac{1}{8}|x| \Rightarrow a = \frac{3}{8}, b = -\frac{1}{8} \Rightarrow ab = -\frac{3}{64}$ ✓

-۷ (۲)

$f_{(x)} = \frac{3x+4}{x+m} \to f_{(2)} = -10 = \frac{6+4}{2+m} \Rightarrow m = -3 \Rightarrow f_{(x)} = \frac{3x+4}{x-3} \Rightarrow f^{-1}_{(x)} = \frac{3x+4}{x-3}$ $(a+d=0 \Rightarrow f_{(x)} = f^{-1}_{(x)})$

$f \circ f^{-1}_{(x)} + f^{-1}_{(x)} = x + \frac{3x+4}{x-3} = x \Rightarrow \frac{3x+4}{x-3} = 0 \Rightarrow x = -\frac{4}{3}$ ✓

-۸ (۲)

$3 - 4x = t \Rightarrow \frac{3-t}{4} = x \Rightarrow \frac{x}{2} = \frac{3-t}{8} \Rightarrow f_{(t)} = 2 - g(\frac{3-t}{4}) \Rightarrow g(\frac{3-t}{4}) = 2 - f_{(t)} \Rightarrow g_{(t)} = 2 - f(3-4t) \Rightarrow$

$\Rightarrow g_{(x)} = 2 - f(3-4x) \xrightarrow{\text{وارون}} g_{(x)} = t \Rightarrow f(3-4x) = 2 - t \Rightarrow 3 - 4x = f^{-1}(2-t) \Rightarrow x = \frac{3 - f^{-1}(2-t)}{4} = g^{-1}_{(t)} \to t = 4 \Rightarrow$

$g^{-1}_{(4)} = \frac{3 - f^{-1}(-2)}{4} \Rightarrow 4g^{-1}_{(4)} = 3 - f^{-1}_{(-2)} \Rightarrow 4g^{-1}_{(4)} + f^{-1}_{(-2)} = 3$ ✓

۹_ قرینه نسبت به مبدأ = قرینه کردن طول و عرض $\Rightarrow -\dfrac{-5x-13}{1+x} = \dfrac{5x+13}{x+1} \longrightarrow \dfrac{5(x-2)+13}{(x-2)+1} - 3 = \longrightarrow$ ۲

$\longrightarrow \dfrac{5x+3}{x-1} - 3 = \dfrac{2x+6}{x-1} = f(x) \to f^{-1}(x) = \dfrac{x+6}{x-2} \to f(x) = f^{-1}(x) \to \dfrac{2x+6}{x-1} = \dfrac{x+6}{x-2} \Rightarrow 2x^2+2x-12 = x^2+5x-6 \Rightarrow$

$\Rightarrow x^2-3x-6=0$ مجموع جوابها $= -\dfrac{b}{a} = \dfrac{-(-3)}{1} = 3$ ✓

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$y = x + 4[x] \Longrightarrow [y] = [x+4[x]] = [x]+4[x] \Rightarrow [y] = 5[x] \Rightarrow [x] = \dfrac{[y]}{5}$ ✓ $\xrightarrow[{[x] = \frac{y-x}{4}}]{\text{صورت سوال}} \dfrac{y-x}{4} = \dfrac{[y]}{5} \Rightarrow$

$5y - 5x = 4[y] \Rightarrow x = \dfrac{5y-4[y]}{5} \Rightarrow f^{-1}(x) = y - \dfrac{4}{5}[y] = g(x) = y = x - \dfrac{4}{5}[x] \Rightarrow f-g(x) = x + 4[x] - x + \dfrac{4}{5}[x] = \longrightarrow$

دقت!

$f-g(x) = \cancel{\dfrac{-x}{4} + 5[x]}$

$\dfrac{24}{5}[x]$