

الف) $y^3 = 2x^2 + 2x - 1$ $\xrightarrow{\text{توان ۳}}$ $y = \sqrt[3]{2x^2 + 2x - 1}$ تابع است.

ب) $x \cos y = y \cos x$ $\xrightarrow{\text{مثال}} x = \frac{\pi}{2}$ $\rightarrow \frac{\pi}{2} \times \cos y = 0 \rightarrow y = \frac{\pi}{2}$ $\rightarrow y = \frac{3\pi}{2}$ تابع نیست.

ج) $2\sqrt{xy} = x + y$ $\xrightarrow{\text{توان ۲}}$ $\{xy = x^2 + 2xy + y^2 \rightarrow x^2 - 2xy + y^2 = 0 \rightarrow (x - y)^2 = 0 \rightarrow x = y\}$ تابع است.

د) $x(y^2 + 2) = y^3 + 2y$ $\rightarrow$ طبق اصول اگر به شکل متقابل باشد $\rightarrow$ تابع است.

$\begin{cases} 2x^2 - b & |x-1| \geq 2 \\ a+2x & -3 \leq x \leq -1 \end{cases} \rightarrow \begin{cases} x-1 \geq 2 \rightarrow x \geq 3 \\ x-1 \leq -2 \rightarrow x \leq -1 \end{cases}$

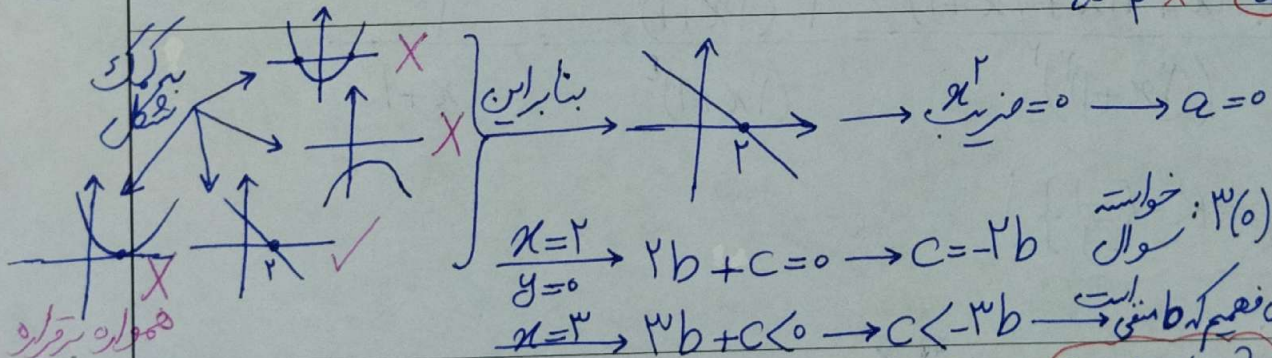
① و ② هر دو معادله $x = -1$ و $x = 3$ می‌کنند.

$x = -1 \rightarrow 2(-1)^2 - b = a + 2(-1) \rightarrow 2 - b = a - 2 \rightarrow a + b = 4$

$b \leq x \leq a \rightarrow (x+b)(x-a) \leq 0$ $\frac{x}{y} \mid \begin{matrix} b & a \\ + & - & - & + \end{matrix}$ یعنی a و b معادله $x^2 + px + q = 0$ را معرفی کنند.

$x = a \rightarrow a - ab - a^2 = 0 \rightarrow b(4b^3 + 2b^2 - 2b) = 0 \rightarrow b = 0 \rightarrow a = 0$ X

$x = b \rightarrow a - b^2 - b^2 = 0 \rightarrow a = 2b^2$ $\rightarrow$ جمع فرایست $\begin{cases} b = -1 \rightarrow a = 2 \\ b = \frac{1}{2} \rightarrow a = \frac{1}{2} \end{cases}$ X $a + b = 1$



الف) $\frac{1}{|x| - [x]} = f(x) \rightarrow |x| \neq [x] \rightarrow R - \{Z^+, 0\}$

ب) $\sqrt{y} = 9 - \sqrt{x}$ $\xrightarrow{\text{توان ۲}}$ $y = x - 18\sqrt{x} + 81 \rightarrow D_f = [0, +\infty)$

الف) $x \neq 1, x > 0, \sqrt[3]{x-1} > 0 \rightarrow x > \frac{1}{3}$

$\log_{\sqrt[3]{x-1}} \geq 0 \rightarrow \log_{\sqrt[3]{x-1}} \geq \log_{\sqrt[3]{x-1}} 1$

$\sqrt[3]{x-1} \geq 1 \rightarrow x \geq \frac{4}{3}$

$\sqrt[3]{x-1} \leq 1 \rightarrow x \leq \frac{1}{3}$

$\frac{1}{4 + \sqrt{|x|} - |x|} > 0 \rightarrow 4 + \sqrt{|x|} - |x| > 0$

$4 + \sqrt{|x|} - |x| \neq 0 \rightarrow x \neq \pm 9$ ①

$t^2 - t - \frac{1}{4} = 0 \rightarrow t = \frac{1 \pm \sqrt{1 + 1}}{2} = \frac{1 \pm \sqrt{2}}{2}$

$x = \pm \frac{1}{4}$

$D_f = (\frac{1}{3}, +\infty) - \{1\}$

سؤال ٦: إذاً نجرب $\rightarrow$ ①, ② $D_f = (-\infty, -\frac{1}{4}) \cup (\frac{1}{4}, +\infty) - \{\pm 9\}$

$f(x) = \begin{cases} \sqrt[3]{x-1} & x > \frac{1}{3} \\ \sqrt[3]{x+1} & x \leq \frac{1}{3} \end{cases}$

$f(x) - x + 1 \geq 0$

$x > \frac{1}{3} \rightarrow \sqrt[3]{x-1} - x + 1 \geq 0 \rightarrow x \geq 0$

$x \leq \frac{1}{3} \rightarrow \sqrt[3]{x+1} - x + 1 \geq 0 \rightarrow x \leq -\frac{1}{3}$

③ $D_f = [0, +\infty)$

$\frac{\sqrt[3]{(1+x^3 + 12x^2 + 9x + 1)}}{(\sqrt[3]{x+1})^2} = \frac{\sqrt[3]{(x+1)^3}}{(\sqrt[3]{x+1})^2} = \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}} = 1$

$D_f = \mathbb{R} - \{-\frac{1}{3}\}$

① $\sqrt[3]{x-a} > 0 \rightarrow x > \frac{a}{3}$

② $1 - \log_{\sqrt[3]{x-a}} \geq 0 \rightarrow \log_{\sqrt[3]{x-a}} \leq \log_{\sqrt[3]{x-a}} 1 \rightarrow \sqrt[3]{x-a} \leq 1 \rightarrow x \leq \frac{10+a}{3}$

$x \in (\frac{a}{3}, \frac{10+a}{3}] \rightarrow c - b = \frac{10}{3} + \frac{a}{3} - \frac{a}{3} = \frac{10}{3}$