

$$g \circ f(x) = \sqrt{-\log_2^{x-1} + 4\sqrt{\log_2^{x-1} - 4}} \quad \text{I) } x-1 > 0 \rightarrow x > 1$$

$$\sqrt{\log_2^{x-1} = t} \quad -t^2 + 4t - 4 \geq 0 \quad \Delta = 0 \quad t = \frac{-4}{-2} = 2 \rightarrow \frac{2}{-6} \rightarrow \{2\}$$

$$\sqrt{\log_2^{x-1} = 2} \rightarrow \log_2^{x-1} = 4 \rightarrow x-1 = 16 \rightarrow x = 17$$

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$$f \circ g(x) = 2(x^2 - 3x + 8) - 5 \rightarrow 2x^2 - 6x + 11$$

$$g \circ f(x) = (2x-5)^2 - 3(2x-5) + 8 = 4x^2 - 20x + 25 - 6x + 15 + 8 = 4x^2 - 26x + 48$$

$$2x^2 - 6x + 11 : 4x^2 - 26x + 48 \rightarrow 2x^2 - 20x + 37$$

$$\alpha^2 + \beta^2 = S^2 - 2P \rightarrow S = 10 \rightarrow 100 - 37 = 63$$

$$P = \frac{37}{2}$$

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$$g(2x) = 5x^2 + 11 \rightarrow g(x) = \frac{5}{4}x^2 + 11 \quad \frac{g(x-7)}{x-7} \rightarrow \frac{5}{4}(x^2 - 14x + 49) + 11$$

$$\frac{5}{4}x^2 - \frac{35}{2}x + \frac{289}{4} \xrightarrow{\text{max}} \frac{35}{2} = \frac{35}{5} = 7 \xrightarrow{x=7} \frac{49 \times 5}{4} - \frac{7 \times 7}{4} + \frac{289}{4} = \frac{44}{4} = 11$$

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$$f \circ g(x) = 3x^2 - 6x - 1 = 3(g(x)) - 4 \rightarrow 3(g(x)) = 3x^2 - 6x + 3$$

$$g(x) = x^2 - 2x + 1$$

$$g(f(x)) = (3x-4)^2 - 2(3x-4) + 1 \rightarrow 9x^2 - 24x + 16 - 6x + 8 + 1 =$$

$$g \circ f(x) = 9x^2 - 30x + 25$$

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$$f - g(x) = \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} \rightarrow m = -1 \quad f - g(x) = -x + 5$$

$$2g(x) - 3f(x) = -14$$

$$(f(x) - g(x)) \times 3 = 3f(x) - 3g(x) = -3x + 15$$

$$-g(x) = -3x + 1$$

$$\left. \begin{array}{l} g(x) = 3x - 1 \\ f(x) = 2x + 4 \end{array} \right\} f \circ g(x) = 2(3x-1) + 4 \rightarrow 6x - 2 + 4 = 6x + 2 = 0$$

$$x = -\frac{1}{3} = a$$

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$$f \circ g(x) \Rightarrow 9^{\log_3 x} = x^{\log_3 9} = x^2$$

$$g \circ f(x) \Rightarrow \log_3^{9x} \Rightarrow \log_3^{2x} = 2x$$

$$x^2 \setminus 2x \rightarrow x^2 - 2x \setminus 0 \rightarrow x(x-2) \setminus 0 \quad \begin{array}{c} 0 \quad 2 \\ + \quad - \quad + \end{array}$$

$$[0, 2] \rightarrow \text{مجموعه}$$

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$$\frac{x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}}{f\left(\frac{x^2 - 2}{x}\right)} \rightarrow \frac{x^4 - 4x^2 + 4}{x^2} + \frac{x^2 - 2}{x} = \frac{(x^2 - 2)^2}{x^2} + \frac{x^2 - 2}{x}$$

$$f(x) = x^2 + x$$

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$$f \circ g(x) = \log_2^{8x - x^2} \quad \text{اولاً: } 8x - x^2 > 0 \rightarrow x(8 - x) > 0 \quad \begin{array}{c} 0 \quad 8 \\ - \quad + \quad - \end{array}$$

$$D_{f \circ g} = (0, 8)$$

$$y: 0 < x < 8 \rightarrow 0 < 8x - x^2 \leq 16 \rightarrow \log_2^{8x - x^2} = y \Rightarrow 2^y = 8x - x^2$$

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$$A = (0, 8)$$

$$B = (-\infty, 4]$$

$$A \cap B = (0, 4] \quad 0 < 2^y \leq 16 \rightarrow -\infty < y \leq 4$$

مجموعه 4

$$f(f(x)) = 1 - 3f^2(x) \rightarrow f(\hat{x}) = 1 - 3x^2$$

$$h \circ g(x) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = 1 - 3x^2$$

$$x^3 + 5x - 3 = 0 \rightarrow \text{معادله درجه 3}$$

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$$g(3) = 8$$

$$f(x) = \frac{x^2 - 4}{x} \rightarrow f(x) = 3 \quad \frac{x^2 - 4}{x} = 3 \rightarrow \frac{x^2 - 4 - 3x}{x} = 0 \rightarrow x^2 - 3x - 4 = 0$$

$$\Delta = 25 \quad x = \frac{3 \pm 5}{2} \rightarrow 4$$

$$x = 4 \rightarrow 64a + 8 = 8 \rightarrow a = 0$$

$$x = -1 \rightarrow -a - 2 = 8 \rightarrow a = -6$$

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