

مسائل جالب / تالیف ۸

$$f(x) = \sqrt{\log_{\frac{1}{2}}(x-1)} \quad g(x) = \sqrt{-2x^2 + 4x - 4} \quad g \circ f(x) \quad (1)$$

$$D_{g \circ f(x)} = \{x \mid x \in D_f, f(x) \in D_g\}$$

$$I: \log_{\frac{1}{2}}(x-1) \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2$$

$$-2x^2 + 4x - 4 \geq 0 \rightarrow b^2 - 4ac = 16 - 16 = 0 \rightarrow x = -\frac{b}{2a} = \frac{-4}{-4} = 1$$

$$D_{g(x)} = \{1\}$$

$$\begin{array}{c} * \\ - \quad | \quad - \end{array}$$

$$II: f(x) \in D_g \rightarrow \sqrt{\log_{\frac{1}{2}}(x-1)} = 1 \rightarrow \log_{\frac{1}{2}}(x-1) = 1 \rightarrow x-1 = \frac{1}{2} \rightarrow x = \frac{3}{2}$$

$$I \cap II: \left\{ \frac{3}{2} \right\} \quad D_{g \circ f(x)} = \left\{ \frac{3}{2} \right\}$$

$$f(x) = vx - \omega \quad g(x) = x^r - vx + 1 \quad f \circ g(x) = g \circ f(x) \quad (r)$$

$$v(x^r - vx + 1) - \omega = (vx - \omega)^r - v(vx - \omega) + 1 \rightarrow$$

$$vx^r - vx + 1 - \omega = vx^r - v_0 x + v\omega - vx + 1\omega + 1 \rightarrow$$

$$vx^r - vx + 1 = vx^r - vx + 1 \rightarrow vx^r - v_0 x + v\omega = 0 \leftarrow \Delta > 0$$

$$B^r + \alpha^r = (\alpha + B)^r - r\alpha B = \left(-\frac{b}{a}\right)^r - r \frac{c}{a} = \left(\frac{v_0}{v}\right)^r - r \left(\frac{v\omega}{v}\right) =$$

$$100 - v\omega = 9v$$

$$g \circ f(x) = \omega x^r + 1 \quad f(x) = vx \quad g(x-v) \quad (r)$$

$$vx = t \rightarrow x = \frac{t}{v} \Rightarrow \omega \left(\frac{t}{v}\right)^r + 1 = \frac{\omega t^r}{v^r} + 1 \rightarrow g(x) = \frac{\omega x^r}{v^r} + 1$$

$$g(x-v) = \frac{\omega (x-v)^r}{v^r} + 1 = \frac{\omega}{v^r} (x^r - rxv + rv^r) + 1 =$$

$$\frac{\omega x^r}{v^r} - \frac{v_0 x}{v^r} + \frac{v\omega}{v^r} \xrightarrow{a>0} \omega x = -\frac{b}{va} = \frac{v_0}{\frac{\omega}{v}} = \frac{1}{v} = V$$

$$x = V \rightarrow \frac{v\omega}{v^r} - \frac{v_0}{v^r} + \frac{v\omega}{v^r} = \frac{v\omega}{v^r} = 1$$

$$f(x) = vx - r \quad f \circ g(x) = vx^r - vx - 1 \quad g \circ f(x) \quad (r)$$

$$v g(x) - r = vx^r - vx - 1 \rightarrow v g(x) = vx^r - vx + v \rightarrow$$

$$g(x) = x^r - vx + 1$$

$$g \circ f(x) = (vx - r)^r - v(vx - r) + 1 = vx^r - vx + 1 - vx + 1 + 1 = vx^r - v_0 x + v\omega$$

$$v g(x) = v f(x) - 1 \quad (r)$$

$$f(x) - g(x) = -x + \omega \xrightarrow{\times v} v f(x) - v g(x) = -vx + v\omega \rightarrow$$

$$v f(x) = -vx + v\omega + v g(x)$$

$$y'g(x) = -y''x + 1 + y'g(x) - 1 \Rightarrow g(x) = y'x - 1$$

$$y'g(x) - y = y'f(x) - 1 \Rightarrow y'f(x) = y'x + 1 \Rightarrow f(x) = x + \frac{1}{y'}$$

$$f \circ g(x) = y'(y'x - 1) + \frac{1}{y'} = y'x + \frac{1}{y'}$$

$$y'x + \frac{1}{y'} = 0 \Rightarrow x = -\frac{1}{y'} = a$$

$$f(x) = a^x \quad g(x) = \log_a x \quad f \circ g(x) \leq g \circ f(x) \quad (9)$$

$$a^{\log_a x} \leq \log_a a^x \Rightarrow x^{\log_a a} \leq x \log_a a \Rightarrow x^1 \leq x \log_a a \Rightarrow x^1 - x \log_a a \leq 0$$

$$\rightarrow x(x - \log_a a) \leq 0 \rightarrow \begin{array}{c} 0 & \log_a a \\ + & - & - & + \end{array} \rightarrow [0, \log_a a] \rightarrow \{0, 1, 2\} \rightarrow x^2$$

$$f\left(\frac{x^2 - y}{x}\right) = x^2 + x - \frac{1}{x} + \frac{1}{x^2} \quad ? f(x) \text{ ist } \dots \quad (10)$$

$$f\left(x - \frac{y}{x}\right) = \left(x - \frac{y}{x}\right)^2 + x - \frac{y}{x} \quad x - \frac{y}{x} = t \rightarrow f(t) = t^2 + t \rightarrow$$

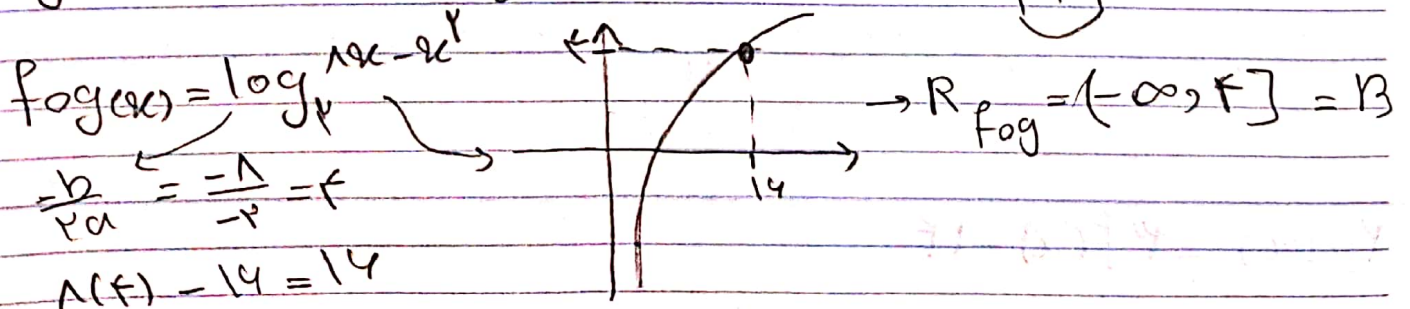
$$f(x) = x^2 + x$$

$$f(x) = \log_a x \quad g(x) = \ln x - x^2 \quad A = D_{f \circ g} \quad B = R_{f \circ g} \quad (11)$$

$$f \circ g(x) = \log_a (\ln x - x^2)$$

$A \cap B?$

$$D_{f \circ g} = \ln x - x^2 > 0 \rightarrow -x(x - \ln x) > 0 \rightarrow \begin{array}{c} 0 & \ln x \\ - & + & + & - \end{array} \rightarrow (0, \ln x) = A$$



$$A \cap B = (0, \frac{1}{2}] \rightarrow \{1, 2, 3, 4\} \rightarrow \text{nicht}$$

$$f \circ f(x) = 1 - x^3 f'(x) \quad g(x) = x - 1 \quad h(x) = x^3 + 2x + 1 \quad (9)$$

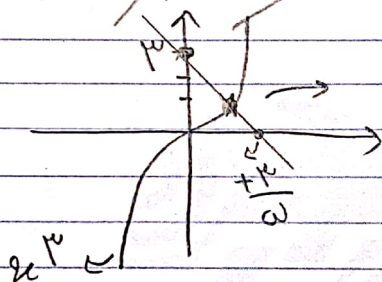
$$h(g(x)) = f(x)$$

$$f(f(x)) = 1 - x^3 f'(x) \xrightarrow{f(x)=t} f(t) = 1 - x^3 t^2 \rightarrow f(x) = 1 - x^3 x^2$$

$$h(g(x)) = f(x) \rightarrow (x-1)^3 + 2(x-1) + 1 = 1 - x^3 x^2 \rightarrow$$

$$x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = 1 - x^3 x^2 \rightarrow x^3 + 5x - 3 = 0$$

$$x^3 = -5x + 3 \rightarrow$$



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$$f(x) = x - \frac{x}{x} \quad g \circ f(x) = ax^3 + 2x \quad g(3) = 1 \quad a? \quad (10)$$

$$x - \frac{x}{x} = 3 \rightarrow \frac{x^2 - x}{x} = 3 \rightarrow x^2 - x = 3x \rightarrow x^2 - 4x = 0 \rightarrow x(x-4) = 0 \rightarrow x = -1 \text{ or } x = 4$$

$$x = -1 \rightarrow g(f(-1)) = -a - 2 = 1 \rightarrow a = -3$$

$$x = 4 \rightarrow g(f(4)) = 4a + 2 = 1 \rightarrow a = -1/4$$