هندسه کرمی — به نام خدا — تکلیف عمومی ۵

(۱)

$f(x) = \sqrt{\log_2^{x-1}}$

$g(x) = \sqrt{-x^2+4x-4}$

$g(x) = \sqrt{-(x-2)^2}$ ⇒ $D_g = \{2\}$

$D_{gof(x)} \{ x \in D_f \;;\; f(x) \in D_g \}$

$D_f : \log_2^{x-1} : \quad x-1>0 \Rightarrow x>1$

و

$\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$ $\Big\} \cap \; \boxed{x \geq 2}$

$f(x) \in D_g \Rightarrow \sqrt{\log_2^{x-1}} = 2 \Rightarrow \log_2^{x-1} = 4 \Rightarrow x-1 = 2^4 = \boxed{x=17}$

$D_{fog} : \quad x \geq 2 \;\cap\; x = 17 \Longrightarrow \boxed{x=17}$ جواب آخر

(۲)

$f(x) = 2x-5$

$g(x) = x^2-3x+8$

$fog(x) = gof(x)$

$2(x^2-3x+8)-5 = (2x-5)^2-3(2x-5)+8$

$2x^2-6x+11 = 4x^2-20x+25-6x+15+8$

$2x^2-6x+11 = 4x^2-26x+48$

$2x^2-20x+37=0$

ریشه‌ها را α و β فرض می‌کنیم ⇐ $\alpha^2+\beta^2 = S^2-2P = 100-2\left(\frac{37}{2}\right) = 63$ جواب آخر

(۳)

$gof(x) = 5x^2+11$

$f(x) = 2x$

$2x = t \to x = \frac{t}{2}$

$g(t) = 5\left(\frac{t^2}{4}\right)+11 \Longrightarrow$

$g(x) = \frac{5x^2}{4}+11$

$\Rightarrow g(x-7) = \frac{5(x-7)^2}{4}+11 \Rightarrow \frac{5}{4}(x^2-14x+49)+11$

$\frac{-b}{2a} = 7 \Rightarrow$ جایگذاری $\frac{5}{4}(49-98+49)+11 = \boxed{11}$ جواب آخر $\Rightarrow \min g(x-7) = \boxed{11}$

(۴)

$f(x) = 3x-4$

$fog(x) = 3x^2-6x-1$

$gof(x) = ?$

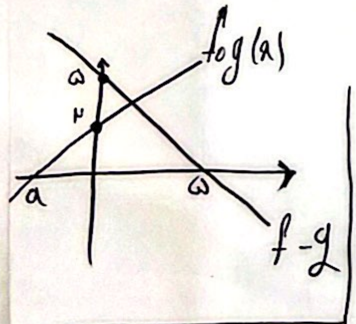
$f(g) = 3g(x)-4 = 3x^2-6x-1 \Rightarrow 3g(x) = 3x^2-6x+3 \Rightarrow$

$g(x) = x^2-2x+1 = (x-1)^2$

$gof(x) = g(f(x)) = ((3x-4)-1)^2 = (3x-5)^2 = 9x^2-30x+25$

جواب آخر

(۵)



$2g(x) = 3f(x)-14 \Rightarrow g(x) = \frac{3}{2}f(x)-7$

$f-g(x) = -x+5 \Rightarrow f(x)-\frac{3}{2}f(x)+7 = -x+5 \Longrightarrow$

$-\frac{1}{2}f(x) = -x-2$

$\boxed{f(x) = 2x+4} \Rightarrow \frac{3}{2}(2x+4)-7 = 3x+6-7 = 3x-1$

$\Rightarrow \boxed{g(x) = 3x-1}$

$fog(x) = 2(3x-1)+4 = 6x+2$

$\Rightarrow 6x+2 = 0 \Rightarrow x = \frac{-2}{6} = \frac{-1}{3}$

جواب آخر $\boxed{a = \frac{-1}{3}}$

(۶)

$fog(x) \leq gof(x)$

$fog(x) = 9^{\log_3 x} = x^{\log_3 9} = x^2$

$\begin{cases} f(x) = 9^x \\ g(x) = \log_3 x \end{cases}$

$gof(x) = \log_3 9^x = x \log_3 9 = 2x$

$x^2 \leq 2x \Rightarrow$

$x^2 - 2x \leq 0$

$x(x-2) \leq 0 \longrightarrow x \in (0, 2]$

	0		2	
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(۷)

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$

$\frac{x^2-2}{x} = x - \frac{2}{x} = t \quad f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x} \Rightarrow f\left(x - \frac{2}{x}\right) = \left(x - \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right)$

$\Rightarrow f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

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(۸)

$f(x) = \log_2 x$ و $g(x) = 8x - x^2$

$A \cap B = ? \longleftarrow \begin{cases} A: D_{fog(x)} \\ B: R_{fog(x)} \end{cases}$

$fog(x) = \log_2^{8x - x^2}$

$\rightarrow D_{fog}: 8x - x^2 > 0 \rightarrow x^2 - 8x < 0$

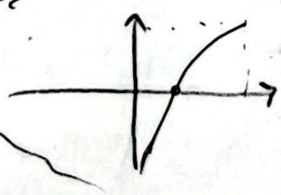
$x(x-8) < 0$ (+ 0 − 8 +)

$x \in (0, 8) = D_{fog} = A = (0, 8)$

$\rightarrow R_{fog} \rightarrow y = 8x - x^2$

$8x - x^2 > 0 \quad \frac{-b}{2a} = \frac{-8}{-2} = 4 \rightarrow y = 32 - 16 = 16$

$R_{\text{عبارت جلو}} = (0, 16] \xRightarrow{\log_2} R_{fog} = (-\infty, 4]$



$D_{fog} = (0, 8) = A$

$R_{fog} = (-\infty, 4] = B$

$\xrightarrow{A \cap B} (0, 4]$

۴ عضو صحیح دارد

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(۹)

$fof(x) = 1 - 3f^2(x) \Rightarrow f(f(x)) = 1 - 3f(x)f(x) \xrightarrow{f(x) = t} f(t) = 1 - 3t^2 \rightarrow f(x) = 1 - 3x^2$

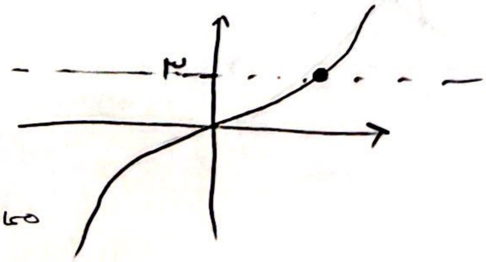
$g(x) = x - 1 , \quad h(x) = x^3 + 2x + 1$

$h(g(x)) = f(x) \longrightarrow (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2$

$\longrightarrow x^3 - 3x(x-1) - 1 + 2x - 1 = 1 - 3x^2$

$x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = 1 - 3x^2$

$x^3 + 5x = 3$



همانطور که می بینیم فقط یک نقطه برخورد دارند پس یک ریشه دارد

(10)

$f(x) = x - \frac{3}{x}$

$g \circ f(x) = ax^3 + 2x$

$g(2) = 8$

$f(x) = 2 \Rightarrow x - \frac{3}{x} = 2$

$x^2 - 3 = 2x \rightarrow x^2 - 2x - 3 = 0$

$\rightarrow (x-3)(x+1) = 0 \nearrow x = -1$

$\searrow x = 3$

$g(2) = ax^3 + 2x = 8 \xrightarrow{x=-1} -a - 2 = 8 \rightarrow -a = 10 \Rightarrow \boxed{a = -10}$

$\searrow x = 2 \quad 8a + 8 = 8 \rightarrow \boxed{a = 0}$