

سوال ۲۰ آفرین !! کتب محاسبات

سوال ۲۰

$$D_{g \circ f(x)} \mid x \in D_f ; f(x) \in D_g \mid$$

$$D_f: \log^{n+1}_r: x-1 > 0 \Rightarrow x > 1$$

$$\log^{n+1}_r 7, 0 \rightarrow x-1 > 1 \rightarrow x > 2$$

$$\boxed{x > 2}$$

$$f(x) = \sqrt{\log^{n+1}_r x-1} \quad (1)$$

$$g(x) = \sqrt{-x^2 + 12x - 5}$$

$$g(x) = \sqrt{-(x-2)^2}$$

$$D_g = \{x \mid x=2\}$$

$$f(x) \in D_g \Rightarrow \sqrt{\log^{n+1}_r x-1} = 2 \Rightarrow \log^{n+1}_r x-1 = 4 \Rightarrow x-1 = r^4 = \boxed{x=17}$$

$$D_{f \circ g}: x > 2 \cap x=17 \Rightarrow \boxed{x=17}$$

$$f(x) = 2x - 5$$

$$g(x) = x^2 - 4x + 11$$

$$f \circ g(x) = g \circ f(x)$$

$$2(x^2 - 4x + 11) - 5 = (2x - 5)^2 - 4(2x - 5) + 11$$

$$2x^2 - 4x + 11 = 4x^2 - 20x + 25 - 4x + 20 + 11$$

$$2x^2 - 4x + 11 = 4x^2 - 24x + 36$$

$$2x^2 - 20x + 25 = 0$$

$$x^2 - 10x + 12.5 = 0 \Rightarrow x = 5 \pm \sqrt{12.5}$$

$$g \circ f(x) = 2x^2 - 4x + 11$$

$$f(x) = 2x$$

$$x = \frac{t}{2}$$

$$g(t) = 2\left(\frac{t}{2}\right)^2 + 11$$

$$g(x) = \frac{2x^2}{2} + 11$$

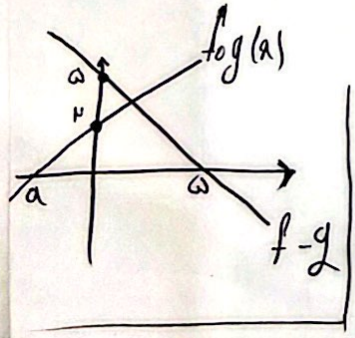
$$\Rightarrow g(x-5) = \frac{2(x-5)^2}{2} + 11 \Rightarrow \frac{2}{2}(x^2 - 10x + 25) + 11$$

$$\frac{-b}{2a} = 5 \Rightarrow \frac{10}{2} = 5 \Rightarrow \boxed{5}$$

$$f \circ g(x) = 2g(x) - 5 = 2(x^2 - 4x + 11) - 5 = 2x^2 - 4x + 11$$

$$g(x) = x^2 - 4x + 11 = (x-2)^2 + 7$$

$$g \circ f(x) = g(f(x)) = (2x-5)^2 + 7 = 4x^2 - 20x + 25 + 7 = 4x^2 - 20x + 32$$



$$2g(x) = 2f(x) - 5 \Rightarrow g(x) = \frac{1}{2}f(x) - \frac{5}{2}$$

$$f - g(x) = -x + 5 \Rightarrow f(x) - \frac{1}{2}f(x) + \frac{5}{2} = -x + 5 \Rightarrow$$

$$f \circ g(x) = 2(2x-5) + 7 = 4x-10+7 = 4x-3$$

$$\Rightarrow 4x-3 = 0 \Rightarrow x = \frac{3}{4}$$

$$\boxed{a = \frac{3}{4}}$$

$$f(x) = 2x + 5$$

$$g(x) = 2x - 1$$

$$\frac{1}{2}(2x+5) - \frac{5}{2} = x + \frac{5}{2} - \frac{5}{2} = x$$

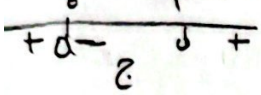
$$f \circ g(x) \leq g \circ f(x) \quad f \circ g(x) = 9^{\log_9 x} = x^{\log_9 9} = x^1 = x \quad \left\{ \begin{array}{l} f(x) = 9^x \\ g(x) = \log_9 x \end{array} \right. \quad (9)$$

$$x^r \leq rx \Rightarrow$$

$$g \circ f(x) = \log_9 9^x = x \log_9 9 = rx \quad (1)$$

$$x^r - rx \leq 0$$

$$x(x-r) \leq 0$$



$$\rightarrow x \in (0, r] \quad \checkmark$$

$$F\left(\frac{x^r-r}{x}\right) = x^r + x - r - \frac{r}{x} + \frac{\varepsilon}{x^r}$$

$$\frac{x^r-r}{x} = x - \frac{r}{x} = t \quad \left\{ \begin{array}{l} F\left(x - \frac{r}{x}\right) = x^r + \frac{\varepsilon}{x^r} - r + x - \frac{r}{x} \Rightarrow F\left(x - \frac{r}{x}\right) = \left(x - \frac{r}{x}\right)^r + \left(x - \frac{r}{x}\right) \end{array} \right. \quad (7)$$

$$\Rightarrow f(t) = t^r + t \Rightarrow f(x) = x^r + x \quad \checkmark$$

$$A \cap B = ? \quad \left\{ \begin{array}{l} A: D_{f \circ g(x)} \\ B: R_{f \circ g(x)} \end{array} \right. \quad \left\{ \begin{array}{l} f(x) = \log_9 x \\ g(x) = \log_9 x \end{array} \right. \quad (1)$$

$$f \circ g(x) = \log_9 \log_9 x$$

$$\begin{aligned} D_{f \circ g} : \log_9 x > 0 &\rightarrow x^r - \log_9 x < 0 \\ x(x-1) < 0 &\rightarrow x \in (0, 1) = D_{f \circ g} = A = (0, 1) \end{aligned} \quad (2)$$

$$\log_9 x > 0 \quad \left\{ \begin{array}{l} \frac{b}{a} = \frac{-1}{-2} = \frac{1}{2} = 2 \end{array} \right. \quad \rightarrow \log_9 x = 2 \quad \rightarrow x = 9^2 = 81$$

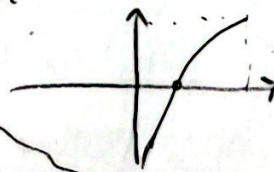
$$R_{f \circ g} = (-\infty, 2] \Rightarrow R_{f \circ g} = (-\infty, 2] \quad \checkmark$$

$$D_{f \circ g} = (0, 1) = A$$

$$R_{f \circ g} = (-\infty, 2] = B$$

$$A \cap B = (0, 1] \quad \checkmark$$

در این مسئله داریم



$$f \circ f(x) = 1 - x^r F(x) \Rightarrow F(F(x)) = 1 - x^r F(x) F(x) \quad f(x) = x \quad F(x) = 1 - x^r \quad (9)$$

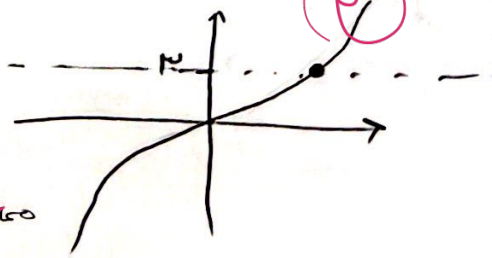
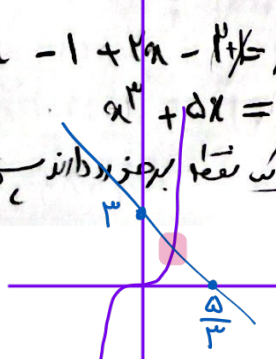
$$h(g(x)) = f(x) \rightarrow (x-1)^r + 2(x-1) + 1 = 1 - x^r \quad \left\{ \begin{array}{l} f(x) = 1 - x^r \\ g(x) = x-1 \end{array} \right. \quad h(x) = x^r + 2x + 1$$

$$\rightarrow x^r - x^r x(x-1) - 1 + 2x - 1 = 1 - x^r$$

$$x^r - x^{r+1} + x^{r+1} - 1 + 2x - 1 = 1 - x^r$$

$$x^r + dx = 3$$

در این مسئله داریم



$$f(x) = 1 \Rightarrow x - \frac{1}{x} = 1$$

$$x^2 - 1 = 1x \rightarrow x^2 - 1x - 1 = 0$$

$$\rightarrow (x-1)(x+1) = 0 \rightarrow x = -1$$

$$\rightarrow x = 1$$

$$g(x) = ax^2 + 1x = 1 \xrightarrow{x=-1} -a - 1 = 1 \rightarrow -a = 1 \Rightarrow \boxed{a = -1}$$

$$x=1 \rightarrow 4a + 1 = 1 \rightarrow \boxed{a = 0}$$

$$f(x) = x - \frac{1}{x}$$

$$g \circ f(x) = ax^2 + 1x$$

$$g(1) = 1$$

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