

پرسش زندگی

$$Dg \circ f = \{x \in D_f \mid f(x) \in D_g\}$$

$$x-1 > 0 \Rightarrow \boxed{x > 1}$$

سوال (1)

$$f(x) = \sqrt{\log_r(x-1)} \rightarrow \log_r(x-1) \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow \boxed{x \geq 2} \quad (I)$$

$$g(x) = \sqrt{-x^2 + 4x - 5} \Rightarrow \sqrt{-(x-2)^2} \Rightarrow Dg = 2$$

$$\sqrt{\log_r(x-1)} = 2 \rightarrow \log_r(x-1) = 4 \Rightarrow (x-1) = 16 \Rightarrow \boxed{x=17} \quad D_{f \circ g}(x) = \{17\}$$

$$f(x) = 2x - 5$$

$$f \circ g = 2(x^2 - 3x + 1) - 5 = 2x^2 - 4x + 14 - 5$$

$$g(x) = x^2 - 3x + 1$$

$$= 2x^2 - 4x + 11$$

$$g(f(x)) = (2x-5)^2 - 3(2x-5) + 1 = \boxed{4x^2 - 14x + 11}$$

$$2x^2 - 4x + 11 = 4x^2 - 14x + 11 \Rightarrow 2x^2 - 10x + 0 = 0$$

$$S = -\frac{b}{a} = 10 \quad P = \frac{c}{a} \Rightarrow S^2 - 10P = 100 - 10\left(\frac{11}{2}\right) = \boxed{45}$$

$$g \circ f(x) = 5x^2 + 11$$

$$f(x) = t \Rightarrow 2x = t \Rightarrow x = \frac{t}{2}$$

سوال (2)

$$f(x) = 2x$$

$$g(x-v) = ?$$

$$g(t) = 5\left(\frac{t}{2}\right)^2 + 11 \Rightarrow g(x) = 5\left(\frac{x}{2}\right)^2 + 11 \Rightarrow g(x) = \frac{5}{4}x^2 + 11$$

$$g(x-v) = \frac{5}{4}(x-v)^2 + 11 \Rightarrow \frac{5(x^2 + 6v - 16x) + 44}{4} = \frac{5x^2 - 10x + 30v + 44}{4}$$

$$\frac{5(v)^2 - 10(v) + 30v + 44}{4} = \boxed{11}$$

$$-\frac{b}{2a} = \frac{v \cdot 10}{10} = v$$

سؤال (٤)

$$f(x) = 3x - 1 \Rightarrow f(g(x)) = 3$$

$$f \circ g(x) = 3x^2 - 4x - 1$$

$$g \circ f = ?$$

$$\Rightarrow 3g(x) - 1 = 3x^2 - 4x - 1$$

$$\Rightarrow 3g(x) = 3x^2 - 4x + 2 \Rightarrow g(x) = x^2 - \frac{4}{3}x + \frac{2}{3}$$

$$g(f(x)) = (3x-1)^2 - \frac{4}{3}(3x-1) + \frac{2}{3} \Rightarrow g(f(x)) = 9x^2 + 4 - 4x + \frac{2}{3} - 4x + \frac{4}{3} + \frac{2}{3}$$

$$= 9x^2 - 8x + \frac{10}{3}$$

سؤال (٥)

$$rg(x) = rf(x) - 1f$$

$$f - g = -x + a \Rightarrow f(x) - g(x) = a - x$$

$$\Rightarrow rf(x) - rg(x) = -1f$$

$$-rf(x) + rg(x) = 1 - rx$$

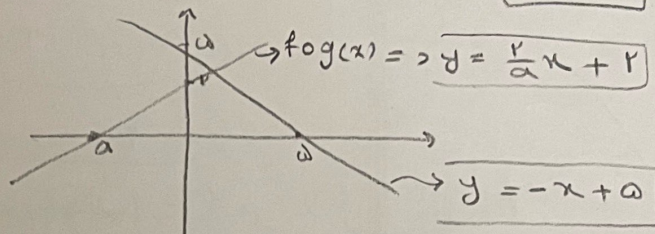
$$rf(x) - rg(x) = 1f$$

$$f(x) = rx + f, g(x) = rx - 1$$

$$f(x) = rx + f \Rightarrow f \circ g(x) = 4x + f$$

$$g(x) = rx - 1$$

$$\frac{r}{a}x + f = 4x + f \rightarrow \frac{r}{a} = 4 \Rightarrow a = \frac{r}{4}$$



$$f(x) = 9^x \quad f \circ g(x) \leq g \circ f(x)$$

$$g(x) = \log_r x \quad f \circ g(x) = 9^{\log_r x} \leq g \circ f(x) = \log_r 9^x$$

$$\Rightarrow 9^{\log_r x} \leq \log_r 9^x \Rightarrow 9^{\log_r x} \leq rx \Rightarrow x^r \leq rx \Rightarrow x^r - rx \leq 0$$

سؤال (٦)

$$\Rightarrow x(x-r) \leq 0 \quad \begin{array}{c|c|c|c} & 0 & r & \\ \hline & + & - & + \\ \hline & x & x & x \end{array}$$

$$f\left(\frac{x^r - r}{x}\right) = x^r + x - f - \frac{r}{x} + \frac{f}{x^r} \quad f(x) = ?$$

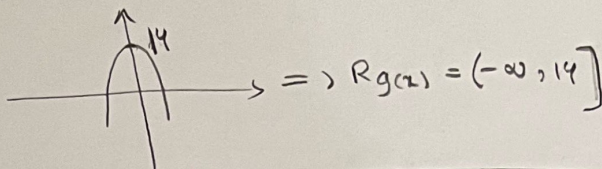
سؤال (٧)

$$\frac{x^r - r}{x} = t \Rightarrow \frac{x^r}{x} - \frac{r}{x} = x - \frac{r}{x} = t \Rightarrow \left(x - \frac{r}{x}\right)^r = t^r \Rightarrow x^r + \frac{f}{x^r} - f = t^r$$

$$f(t) = x^r + \frac{f}{x^r} - f + x - \frac{r}{x} = t^r + t \Rightarrow f(x) = x^r + x$$

$$f(x) = \log_r x \Rightarrow D_f = \{x > 0\}$$

$$g(x) = 11x - x^2 \Rightarrow D_g = \mathbb{R}$$



سوال ۱۷

$$A = D_{f \circ g}(x) \Rightarrow D_{f \circ g} = \{x \in D_g \mid g(x) \in D_f\} \Rightarrow D_{f \circ g} = (0, 11) = A$$

$$B = R_{f \circ g}(x)$$

$$A \cap B = ?$$

$$(II) x > 0$$

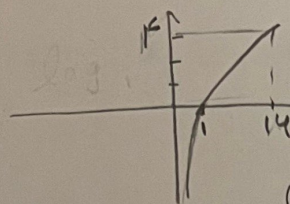
$$(I) 11x - x^2 > 0 \Rightarrow x^2 - 11x < 0$$

$$x(x - 11) < 0$$

$$\begin{array}{c} 0 \quad 11 \\ + \quad - \quad + \\ \hline \Rightarrow (0, 11) \end{array}$$

g(x) است (بر) Rg = (-infinity, 14]

سین بر غلظت
دایره f در قطر
مستقیم



$$R_{f \circ g} = (-\infty, 14] = B$$

$$A \cap B = (0, 11) \cap (-\infty, 14] = (0, 11] \Rightarrow \text{شماره های صحیح از ۱ تا ۱۱ است}$$

$$f \circ f(x) = 1 - 3f'(x)$$

$$h \circ g(x) = (x-1)^2 + 2(x-1) + 1 \Rightarrow$$

$$g(x) = x-1$$

$$x^2 - 3x^2 + 3x - 1 + 2x - 2 + 1 = x^2 - 3x^2 + 5x - 2$$

$$h(x) = x^2 + 2x + 1$$

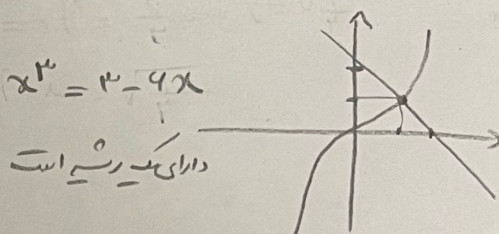
$$f(f(x)) = 1 - 3f'(x)$$

$$h(g(x)) = f(x)$$

$$f(x) = t$$

$$\Rightarrow f(t) = 1 - 3t^2 \Rightarrow f(x) = 1 - 3x^2$$

$$x^2 - 3x^2 + 5x - 2 = 1 - 3x^2 \Rightarrow x^2 + 5x - 3 = 0 \Rightarrow x^2 = 3 - 5x$$



دوای یک ریشه است

سوال ۱۷

$$f(x) = x - \frac{f}{x}$$

$$x - \frac{f}{x} = 3 \Rightarrow \frac{x^2 - f}{x} = 3 \Rightarrow x^2 - 3x - f = 0$$

$$g \circ f(x) = ax^2 + 2x$$

$$\Rightarrow x = -1 \leq x = 3$$

$$g(3) = 11$$

$$a = ?$$

$$g \circ f(-1) = -a - 2 = 11 \Rightarrow a = -13$$

$$g \circ f(3) = 9a + 11 = 11 \Rightarrow a = 0$$

$$\Rightarrow a = 0 \leq -13$$