

19, 75

پرسش زندگی

$$Dg \circ f = \{x \in D_f \mid f(x) \in D_g\}$$

$$x-1 > 0 \Rightarrow \boxed{x > 1}$$

سوال 1

$$f(x) = \sqrt{\log_r(x-1)} \rightarrow \log_r(x-1) \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow \boxed{x \geq 2} \quad (I)$$

$$g(x) = \sqrt{-x^2 + 4x - 5} \Rightarrow \sqrt{-(x-2)^2} \Rightarrow Dg = 2$$

2

$$\sqrt{\log_r(x-1)} = r \rightarrow \log_r(x-1) = r \Rightarrow (x-1) = 14 \Rightarrow \boxed{x = 15} \quad D_{f \circ g}(x) = \{15\}$$

$$f(x) = 2x - 5$$

$$f \circ g = r(x^2 - 3x + 1) - 5 = 2x^2 - 4x + 14 - 5$$

$$g(x) = x^2 - 3x + 1$$

$$= 2x^2 - 4x + 11$$

$$g(f(x)) = (2x - 5)^2 - r(2x - 5) + 1 = \boxed{4x^2 - 24x + 11}$$

$$2x^2 - 4x + 11 = 4x^2 - 24x + 11 \Rightarrow 2x^2 - 20x + 20 = 0$$

$$S = -\frac{b}{a} = 10 \quad P = \frac{c}{a} = 1 \Rightarrow S^2 - 10P = 100 - 10 = 90$$

$$g \circ f(x) = 5x^2 + 11$$

$$f(x) = t \Rightarrow 2x = t \Rightarrow x = \frac{t}{2}$$

سوال 3

$$f(x) = 2x$$

$$g(x-v) = ?$$

$$g(t) = 5\left(\frac{t}{2}\right)^2 + 11 \Rightarrow g(x) = 5\left(\frac{x}{2}\right)^2 + 11 \Rightarrow g(x) = \frac{5}{4}x^2 + 11$$

$$g(x-v) = \frac{5}{4}(x-v)^2 + 11 \Rightarrow \frac{5(x^2 + 6v - 16x) + 44}{4} = \frac{5x^2 - 20x + 36v + 44}{4}$$

$$\frac{5(v)^2 - 20(v) + 36v + 44}{4} = 11$$

$$-\frac{b}{2a} = \frac{v}{10} = v$$

سؤال (٤)

$$f(x) = 3x - 1 \Rightarrow f(g(x)) = 3$$

$$f \circ g(x) = 3x^2 - 4x - 1$$

$$g \circ f = ?$$

$$\begin{aligned} &\Rightarrow 3g(x) - 1 = 3x^2 - 4x - 1 \\ &\Rightarrow 3g(x) = 3x^2 - 4x + 2 \Rightarrow g(x) = x^2 - \frac{4}{3}x + \frac{2}{3} \end{aligned}$$

$$g(f(x)) = (3x - 1)^2 - \frac{4}{3}(3x - 1) + \frac{2}{3} \Rightarrow g(f(x)) = 9x^2 + 4 - 4x + \frac{2}{3} - 4x + \frac{4}{3} + \frac{2}{3}$$

$$= 9x^2 - 8x + \frac{10}{3}$$

سؤال (٥)

$$rg(x) = rf(x) - 1f$$

$$f - g = -x + a \Rightarrow f(x) - g(x) = a - x$$

$$\Rightarrow rf(x) - rg(x) = -1f$$

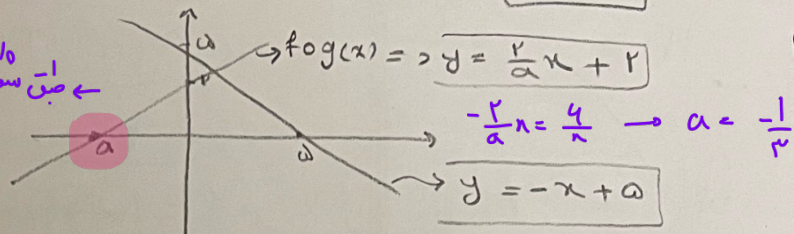
$$-rf(x) + rg(x) = 1 - 3x$$

$$rf(x) - rg(x) = 1f$$

$$f(x) = 3x + 1, g(x) = 3x - 1$$

$$\begin{aligned} f(x) &= 3x + 1 \Rightarrow f \circ g(x) = 4x + 1 \\ g(x) &= 3x - 1 \end{aligned}$$

$$\frac{3}{2}x + 1 = 4x + 1 \rightarrow \frac{3}{2} = a \Rightarrow a = \frac{3}{2}$$



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$$f(x) = 9^x \quad f \circ g(x) \leq g \circ f(x)$$

$$g(x) = \log_9 x \quad f \circ g(x) = 9^{\log_9 x} \leq g \circ f(x) = \log_9 9^x$$

$$\Rightarrow 9^{\log_9 x} \leq \log_9 9^x \Rightarrow x^{\log_9 x} \leq x \Rightarrow x^r \leq x \Rightarrow x^r - x \leq 0$$

$$\Rightarrow [0, 1] \rightarrow 0, 1, 2$$

$$[0, 2]$$

$$\Rightarrow x(x - 1) \leq 0$$

$$f\left(\frac{x^r - r}{x}\right) = x^r + x - r - \frac{r}{x} + \frac{r}{x^r} \quad f(x) = ?$$

$$\frac{x^r - r}{x} = t \Rightarrow \frac{x^r}{x} - \frac{r}{x} = x - \frac{r}{x} = t \Rightarrow (x - \frac{r}{x})^r = t^r \Rightarrow x^r + \frac{r}{x^r} - r = t^r$$

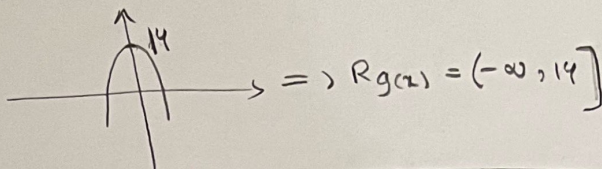
$$f(t) = x^r + \frac{r}{x^r} - r + x - \frac{r}{x} = t^r + t \Rightarrow f(x) = x^r + x$$

سؤال (٧)

1/10

$$f(x) = \log_r x \Rightarrow D_f = \{x > 0\}$$

$$g(x) = 1 - x^2 \Rightarrow D_g = \mathbb{R}$$



سوال ۱

$$A = D_{f \circ g}(x) \Rightarrow D_{f \circ g} = \{x \in D_g \mid g(x) \in D_f\} \Rightarrow D_{f \circ g} = (0, 1) = A$$

$$B = R_{f \circ g}(x)$$

$$A \cap B = ?$$

$$(II) x > 0$$

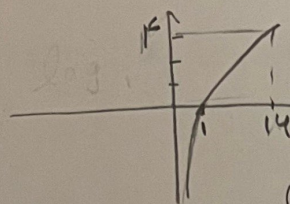
$$(I) 1 - x^2 > 0 \Rightarrow x^2 - 1 < 0$$

$$x(x-1) < 0$$

$$\begin{matrix} 0 & 1 \\ + & - & + \end{matrix} \Rightarrow (0, 1)$$

$$\begin{matrix} x=0 \\ x=1 \end{matrix}$$

گراف (x) را بر R_g = (-∞, 14] رسم کنید



سینوس بر غلظت
دایره f در قطر
مستقیم

$$R_{f \circ g} = (-\infty, 14] = B$$

$$A \cap B = (0, 1) \cap (-\infty, 14] = (0, 1) \Rightarrow \text{محل اعداد صحیح از ۰ تا ۱ است}$$

$$f \circ f(x) = 1 - r f'(x)$$

$$h \circ g(x) = (x-1)^2 + r(x-1) + 1 \Rightarrow$$

$$g(x) = x-1$$

$$x^2 - rx^2 + rx - 1 + rx - r + 1 = x^2 - rx^2 + 2rx - r$$

$$h(x) = x^2 + rx + 1$$

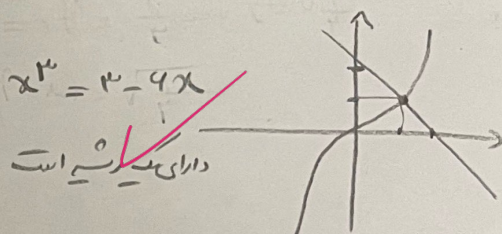
$$f(f(x)) = 1 - r f'(x)$$

$$h(g(x)) = f(x)$$

$$f(x) = t$$

$$\Rightarrow f(t) = 1 - rt^2 \Rightarrow f(x) = 1 - rx^2$$

$$x^2 - rx^2 + 2rx - r = 1 - rx^2 \Rightarrow x^2 + 2rx - r = 0 \Rightarrow x^2 = r - 4x$$



دایره f در قطر است

سوال ۱۶

$$f(x) = x - \frac{f}{x}$$

$$x - \frac{f}{x} = r \Rightarrow \frac{x^2 - f}{x} = r \Rightarrow x^2 - rx - f = 0$$

$$g \circ f(x) = ax^2 + rx$$

$$\Rightarrow x = -1 \leq x = r$$

$$g(r) = 1 -$$

$$a = ?$$

$$g \circ f(-1) = -a - r = 1 \Rightarrow a = -1$$

$$g \circ f(r) = 4a + 1 = 1 \Rightarrow a = 0$$

$$\Rightarrow a = 0 \leq -1$$