① $D_{gof_{(x)}} = D_f \cap f(x) \in D_g$

$D_f = x-1>0 \Rightarrow x>1$ ∩ $x \geq 2$

$\log_2^{x-1} \geq 0$, $x-1 \geq 1 \Rightarrow x \geq 2$

$D_g = -(x^2-4x+4) \geq 0$, $D_g = \{2\}$

$-(x-2)^2 \geq 0$

$\sqrt{\log_2^{x-1}} = 2$, $\log_2^{x-1} = 4$, $x-1=16$, $x=17$

$x \geq 2 \cap x = 17$, $D_{gof} = \{17\}$

② $f(x^2-3x+8) = g(2x-5)$

$2(x^2-3x+8)-5 = (2x-5)^2 - 4(2x-5) + 8$

$2x^2-6x+16-5 = 4x^2-20x+25-6x+15+8$

$0 = 2x^2-20x+37$

$S^2-2P = 100-2\left(\frac{37}{2}\right) = 63$

$S = \frac{20}{2} = 10$

$P = \frac{37}{2}$

③ $g(2x) = 5x^2+11$, $2x = t$, $g(t) = \frac{5}{4}t^2+11$

$g(x-7) = \frac{5}{4}(x-7)^2+11 = \frac{5}{4}x^2 - \frac{70}{4}x + \frac{245}{4} + 11$

$= \frac{5}{4}x^2 - \frac{70}{4}x + \frac{289}{4}$, $\frac{-b}{2a} = \frac{\frac{70}{4}}{\frac{10}{4}} = 7$

$\xrightarrow{x=7}$ $\frac{5\times 49}{4} - \frac{490}{4} + \frac{289}{4} = \frac{44}{4} = 11$

سارا عبداللهی

(۴)

$$f(g(x)) = 3g(x) - 4 = 3x^2 - 6x - 1$$

$$g(x) = \frac{3x^2 - 6x + 3}{3} = x^2 - 2x + 1 = (x-1)^2$$

$$gof(x) = g(f(x)) = g(3x-4) = (3x-4-1)^2 = (3x-5)^2 = 9x^2 - 30x + 25$$

(۵)

$f-g(x) = -x+5 \qquad f(x) - g(x) = -x+5$

$g(x) = \frac{3f(x)-14}{2} \qquad \Downarrow \quad f(x) - \left(\frac{3f(x)-14}{2}\right) = -x+5$

$\rightarrow \; 2f(x) - 3f(x) + 14 = -2x + 10$

$2x+4 = f(x)$

$$g(x) = \frac{3(2x+4)-14}{2} = \frac{6x-2}{2} = 3x-1$$

$f(g(x)) = \frac{2}{a}x + 2 \qquad f(g(x)) = f(3x-1) = 2(3x-1)+4 = 6x+2$

$\frac{2}{a}x + 2 = 6x + 2 \qquad \frac{2}{a} = 6 \qquad a = \frac{1}{3}$

(۶)

$f(g(x)) = 9^{\log_3 x} = x^2 \qquad g(f(x)) = \log_3 9^x = 2x$

$x^2 \le 2x \qquad x^2 - 2x \le 0 \qquad \begin{array}{ccc} & 0 & 2 \\ \hline + & 0 \; - & 0 \; + \end{array} \qquad 0 \le x \le 2$

او۲ ← دوعضو $\qquad 0 < x \le 2 \;\Leftarrow\; (\log_3 0)$ غیرقابل قبول ۰

(۷)

$$f\left(x - \frac{2}{x}\right) = x - \frac{2}{x} + \left(x^2 + \frac{4}{x^2} - 4\right) = x - \frac{2}{x} + \left(x - \frac{2}{x}\right)^2$$

$$f(x) = x + x^2$$

$$D_{fog} = Dg \cap g(x) \in Df$$

$$\mathbb{R} \cap \quad \wedge x - x^2 > 0$$

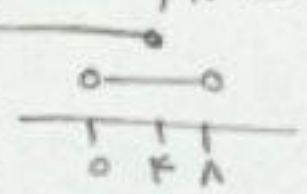
$$Dfog = 0 < x < 8 = A$$

$$\star_{8x-x^2} = x(-x+8)$$

$$\max = 32 - 16 = 16$$

$$fog(x) = \log_2^{8x-x^2} \xrightarrow{\star} \log_2^{16} = 4$$

$$R_{fog} = (-\infty, 4] = B \qquad A \cap B = (0, 4]$$



$$f(f(x)) = 1 - 3f^2(x) \longrightarrow f(x) = 1 - 3x^2$$

(۹)

$$h(x-1) = 1 - 3x^2$$

$$(x-1)^3 + 2(x-1) + 1 = 1 - 3x^2$$

$$x^3 - 3x^2 + 3x + 1 + 2x - 2 + 1 = 1 - 3x^2$$

$$x^3 + 5x - 1 = 0$$

با استفاده از شکل یک جواب دارد

(۱۰)

$$g(f(x)) = g\left(x - \frac{4}{x}\right) = ax^3 + 2x$$

$$x - \frac{4}{x} = 3 \qquad g\left(-1 - \frac{4}{-1}\right) = -a - 2 = 8$$

$$-10 = a$$

$$x = -1$$

$$x = 4$$

$$g\left(4 - \frac{4}{4}\right) = 64a + 8 = 8$$

$$a = 0$$