نام: دیبا احمدی   تکلیف شماره ۵   مقطع: دوازدهم ریاضی B   ۱۹ آزمون ۱۱

①

$D_{gof} = \{x \in D_f , f(x) \in D_g\} \to D_f : \begin{cases} x-1>0 \to x>1 \\ \log_2^{x-1} \geq 0 \to x-1 \geq 2^0 \to x \geq 2 \end{cases} \to D_f = [2,+\infty)$ ①

۱/۲۵

$D_g = 2$ دقت کن

$D_g \Rightarrow -x^2+4x-4 \geq 0 \to \frac{2}{-\phi-} \to D_g = \{\cancel{x}\}$ (2) $\to f(x) \in D_g \to \sqrt{\log_2^{x-1}} = 2 \to$ ②

$D_{gof} = ① \cap ② = \boxed{\{\cancel{x}\}}$ 17 $\sqrt{\log_2^{x-1}} = 2 \to \log_2^{x-1} = 4 \to x-1 = 16 \to x = 17$

②

$fog = 2(x^2-3x+8)-5 = 2x^2-6x+11$

$gof = (2x-5)^2 - 3(2x-5)+8 = 4x^2-26x+48$

$gof = fog \Rightarrow 2x^2-20x+37 = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{20 \pm \sqrt{20^2-4(2)(37)}}{4}$ $\nearrow x_1 = \frac{10+\sqrt{26}}{2}$, $\searrow x_2 = \frac{10-\sqrt{26}}{2}$ $\to x_1^2+x_2^2 = \frac{200+52}{4} = \boxed{63}$

۲

③

$gof = g(2x) = 5x^2+11 \to 2x = t \to x = \frac{t}{2} \to g(t) = 5(\frac{t}{2})^2+11 = \frac{5t^2}{4}+11$

$\to g(x) = \frac{5x^2}{4}+11 \to g(x-7) = \frac{5(x-7)^2}{4}+11 = \frac{5}{4}x^2 - \frac{70}{4}x + \frac{289}{4} \to \min \left| \frac{-b}{2a} = 7 \right. \quad \boxed{11}$

۲

④

$fog = 3g-4 = 3x^2-6x-1 \to 3g = 3x^2-6x+3 \to g = x^2-2x+1$

$gof = (3x-4)^2 - 2(3x-4)+1 = \boxed{9x^2-30x+25}$

۲

⑤

$g(x) = ax+b$, $f(x) = cx+d$ $\to 2g = 3f-14 \to 2ax+2b = 3cx+3d-14 \to \begin{cases} 2a = 3c \to a = \frac{3}{2}c \; * \\ 2b = 3d-14 \to b = \frac{3d-14}{2} \end{cases}$

$\to f-g = cx+d-ax-b = (c-a)x+d-b \xrightarrow{(0,5)} d-b = 5 \to d - \frac{3d-14}{2} = 5 \to d = 4$

$b = -1$

$\xrightarrow{(5,0)} 5(c-a)+4+1 = 0 \to c-a = -1 \xrightarrow{*} c - \frac{3}{2}c = -1 \to c = 2 \to a = 3$

$\left. \begin{matrix} g(x) = 3x-1 \\ f(x) = 2x+4 \end{matrix} \right\} \to fog = 2(3x-1)+4 = 6x+2 \xrightarrow{(a,0)} 6a+2 = 0 \to 6a = -2 \to \boxed{a = -\frac{1}{3}}$

۲

⑥

$fog \leq gof \to 9^{\log_3^x} \leq \log_3^{9^x} \to x^{\log_3^9} \leq 2x\log_3^3 \to x^2 \leq 2x \to x^2-2x \leq 0 \to \frac{0 \quad 2}{+ \; 0 \; - \; 0 \; +}$

۲

$\xrightarrow{D_g = (0,+\infty)} x \in (0,2] \to$ ۲ عدد صحیح

⑦

$f(\underbrace{x-\frac{2}{x}}_{t}) = \underbrace{x^2-4+\frac{4}{x^2}}_{(x-\frac{2}{x})^2} + \underbrace{x-\frac{2}{x}}_{x-\frac{2}{x}} \to f(t) = t^2+t \to f(x) = x^2+x$

۲

۸

$D_{fog} = \{x \in D_g \,,\, g(x) \in D_f\} \to D_g = \mathbb{R}$ ① , $D_f \Rightarrow x > 0 \to 8x - x^2 > 0 \to$ $\begin{array}{ccccc} & 0 & & 8 & \\ - & & + \; + & & - \end{array}$

$x \in (0,8)$ ② $\to$ ①∩② $= D_{fog} = (0,8) = A$

$fog = \log_2 (8x - x^2) \xrightarrow{R_{fog}} \max \log_2 (8x - x^2) \to \max \left| \begin{array}{l} \frac{-b}{2a} = 4 \\ 16 \end{array} \right. \to \log_2 16 = 4 \} \to R_{fog} = (0,4] = B$

۲

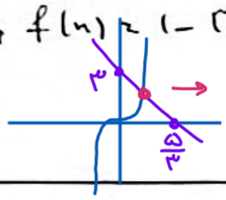
$B \cap A = (0,4] \to$ $\boxed{\text{۴ عدد صحیح}}$ $\xrightarrow{D_{fog}} Min = 0 \to$ چون ۰ در این بازه قرار نمی‌گیرد پس بازه باز (۰,۴] داریم.

۹

۱,۷۵

$f(\underbrace{f(x)}_{t}) = 1 - 2f^2(x) = 1 - 2t^2 \to f(x) = 1 - 2x^2$

$\log_2 (x-1)^2 + 2(x-1) + 1$



یک ریشه $\} \to f(x) = \log(x) \to$ $\boxed{x^3 + 5x - 3 = 0}$

تعداد جواب؟ 5

۱۰

$g(3) = 8 \to f(x) = 3 \to x - \frac{4}{x} = 3 \to x^2 - 3x - 4 = 0$ $\begin{cases} x = -1 \\ x = 4 \end{cases}$

$gof = \begin{cases} \xrightarrow{x=-1} -a = 10 \to a = \boxed{-10} \\ \xrightarrow{x=4} 4a + 8 = 8 \to a = \boxed{0} \end{cases}$

۲