تیم بهار

تکلیف شماره ۸

-۱

$$f(x)=\sqrt{\log_{۲}^{(x-۱)}} \rightarrow D_f \rightarrow \log_{۲}^{x-۱} \geq ۰ \rightarrow x-۱\geq ۱ \rightarrow x\geq ۲$$

$$x-۱>۰ \rightarrow x>۱$$

$$\left.\right\} x \geq ۲$$

$$g(x)=\sqrt{-x^۲+۴x-۴} \longrightarrow -x^۲+۴x-۴\geq ۰ \longrightarrow x^۲-۴x+۴\leq ۰ \longrightarrow (x-۲)^۲\leq ۰$$

$$\boxed{x=۲}$$

$$D_{gof(x)}=\{x \,/\, \underbrace{x\in D_f}_{\text{I}\; x\geq ۲} \;,\; \underbrace{f(x)\in D_g}_{\sqrt{\log_{۲}^{x-۱}}=۲ \,\rightarrow\, \log_{۲}^{x-۱}=۴ \,\rightarrow\, x-۱=۱۶}\}$$

$$\text{II} \quad x=۱۷$$

$$\text{I}\cap\text{II} \longrightarrow x=۱۷ \qquad D_{gof(x)}=\{۱۷\}$$

-۲

$$f(x)=۲x-۵$$

$$g(x)=x^۲-۳x+۸$$

$$fog(x)=gof(x) \longrightarrow ۲(x^۲-۳x+۸)-۵=\overbrace{(۲x-۵)^۲-۳(۲x-۵)+۸}^{۴x^۲+۲۵-۲۰x-۶x+۱۵+۸}$$

$$\longrightarrow ۲x^۲-۶x+۱۶-۵=۴x^۲-۲۶x+۴۸ \longrightarrow ۲x^۲-۲۰x+۳۷=۰$$

$$\alpha+\beta=\frac{-b}{a}=\frac{۲۰}{۲}=۱۰$$

$$\alpha\beta=\frac{c}{a}=\frac{۳۷}{۲}$$

$$\alpha^۲+\beta^۲=(\alpha+\beta)^۲-۲\alpha\beta=۱۰۰-۲\left(\frac{۳۷}{۲}\right)=$$

$$۱۰۰-۳۷=۶۳$$ جواب

-۳

$$gof(x)=۵x^۲+۱۱$$

$$f(x)=۲x \longrightarrow f(x)=t \rightarrow ۲x=t \rightarrow x=\frac{t}{۲}$$

$$g(t)=۵\left(\frac{t}{۲}\right)^۲+۱۱=\frac{۵t^۲}{۴}+۱۱ \longrightarrow g(x)=\frac{۵x^۲}{۴}+۱۱$$

$$g(x-۷)=\frac{۵(x-۷)^۲}{۴}+۱۱=\frac{۵(x^۲-۱۴x+۴۹)}{۴}+۱۱=\frac{۵x^۲-۷۰x+۲۸۹}{۴}$$

$$\longrightarrow \left| \begin{array}{l} \dfrac{-b}{۲a}=\dfrac{۷۰}{۱۰}=۷ \\[2ex] \dfrac{۵(۷)^۲-۷۰(۷)+۲۸۹}{۴}=\dfrac{۲۴۵-۴۹۰+۲۸۹}{۴}=۱۱ \end{array} \right.$$ جواب

جواب

۴-

$f(x) = 3x - 4 \longrightarrow f(g(x)) = 3g(x) - 4$

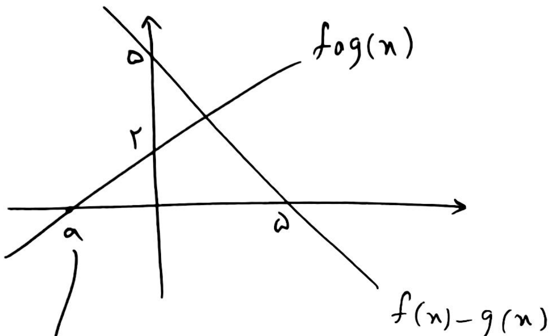
$fog(x) = 3x^2 - 6x - 1$

$\longrightarrow 3g(x) - 4 = 3x^2 - 6x - 1 \longrightarrow 3g(x) = 3x^2 - 6x + 3 \longrightarrow$

$g(x) = x^2 - 2x + 1 = (x-1)^2$

$gof(x) = (3x - 4 - 1)^2 = (3x - 5)^2 = 9x^2 - 30x + 25$ جواب

۵-



$f(x) - g(x) = -x + 5$

$2g(x) = 3f(x) - 14$

$fog(x) = \frac{2}{a}x + 2$

$a < 0$

$\longrightarrow \begin{cases} f(x) - g(x) = 5 - x \xrightarrow{\times -2} \\ 3f(x) - 2g(x) = 14 \longrightarrow \end{cases} \begin{cases} -2f(x) + \cancel{2g(x)} = 2x - 10 \\ 3f(x) - \cancel{2g(x)} = 14 \end{cases}$

$\boxed{f(x) = 2x + 4}$

$g(x) = f(x) - 5 + x = \boxed{3x - 1}$

$fog = 2(3x - 1) + 4 = 6x + 2$

$6x + 2 = \frac{2}{a}x + 2 \longrightarrow \frac{2}{a} = 6 \longrightarrow a = \frac{1}{3} \xrightarrow{a<0} a = \frac{-1}{3}$ جواب

بقیه جواب‌ها

۶ـ

$f(x) = 9^x \qquad f\circ g(x) \leq g\circ f(x)$

$g(x) = \log_3^x$

$f\circ g(x) = 9^{\log_3^x} = x^{\log_3^9} = x^2 \rightarrow f\circ g(x) = x^2$

$g\circ f(x) = \log_3^{9^x} = x\log_3^9 = 2x \rightarrow g\circ f(x) = 2x$

$\rightarrow x^2 \leq 2x \rightarrow x^2 - 2x \leq 0 \rightarrow x(x-2) \leq 0$

$\begin{array}{c|c|c} & 0 & & 2 & \\ + & & - & & + \end{array}$

$\rightarrow [0,2] = \{0, 1, 2\} \rightarrow$ شامل ۳ عدد صحیح است.

۷ـ

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$

$f(\overbrace{x - \tfrac{2}{x}}^{t}) = \overbrace{(x - \tfrac{2}{x})^2}^{t^2} + \overbrace{x - \tfrac{2}{x}}^{t} \longrightarrow f(x) = x^2 + x$

$(x - \frac{2}{x})^2 = x^2 + \frac{4}{x^2} - 4$

۸ـ

$f(x) = \log_2^x \rightarrow x > 0 \qquad D_{f\circ g} \rightarrow A \qquad R_{f\circ g} \rightarrow B \qquad A \cap B = ?$

$g(x) = 8x - x^2 \rightarrow \mathbb{R}$

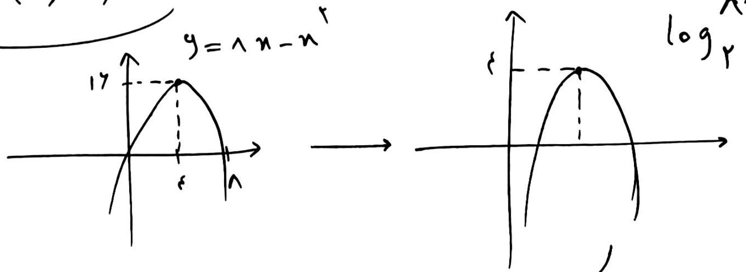
$D_{f\circ g} : \{x \,/\, \underbrace{x \in D_g}_{\mathrm{I}\ \mathbb{R}},\ \underbrace{g(x) \in D_f}\}$

$8x - x^2 > 0 \rightarrow x^2 - 8x < 0 \qquad \begin{array}{c|c|c} & 0 & & 8 & \\ + & & - & & + \end{array} \rightarrow (0, 8)\ \mathrm{II}$

$\mathrm{I} \cap \mathrm{II} \rightarrow (0,8) \longrightarrow A = (0,8)$

$f\circ g(x) = \log_2^{8x - x^2}$

$D_{f\circ g(x)} = (0, 8)$



$B = (-\infty, 4] \longleftarrow R_{f\circ g(x)} = (-\infty, 4]$

$A \cap B = (0, 4] = \{1, 2, 3, 4\}$ جواب ← شامل ۴ عضو صحیح است

[illegible]

۹-

$f \circ f(x) = 1 - 3f^2(x)$

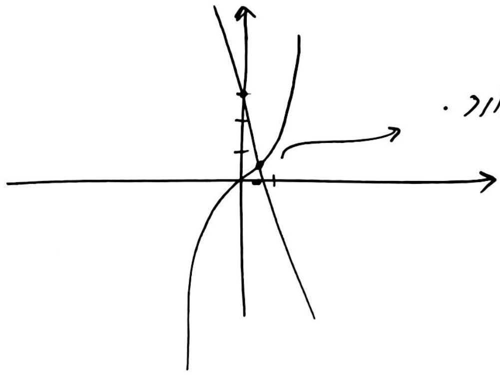
$f(x) = t \longrightarrow f(t) = 1 - 3t^2 \longrightarrow \boxed{f(x) = 1 - 3x^2}$

$\left.\begin{aligned} g(x) &= x - 1 \\ h(x) &= x^3 + 2x + 1 \end{aligned}\right\} \quad h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 1 + 3x - 3x^2 + 2x - 2 + 1$

$= x^3 - 3x^2 + 5x - 2$

$\longrightarrow h(g(x)) = f(x) \longrightarrow x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2}$

$\longrightarrow x^3 + 5x - 2 = 1 \longrightarrow x^3 = 3 - 5x$



1 ریشه دارد.

۱۰-

$f(x) = x - \frac{4}{x}$

$g \circ f(x) = ax^3 + 2x$

$g(3) = 8$

$\downarrow$

$f(x) = 3 \longrightarrow x - \frac{4}{x} = 3 \longrightarrow \frac{x^2 - 4}{x} = 3 \longrightarrow x^2 - 3x - 4 = 0 \longrightarrow (x-4)(x+1) = 0$

$x = 4$ یا -1

if $x = 4 \longrightarrow a(4)^3 + 2(4) = 8 \longrightarrow 64a = 0 \longrightarrow \boxed{a = 0}$

if $x = -1 \longrightarrow a(-1)^3 + 2(-1) = 8 \longrightarrow -a = 10 \longrightarrow \boxed{a = -10}$

جواب $a = -10$ یا 0