تکلیف شماره ۸ — نیکا بهاری

۲۰ آفرین !!

۱-

$f(x) = \sqrt{\log_2^{(x-1)}} \rightarrow D_f \rightarrow \log_2^{x-1} \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2$

$x-1>0 \rightarrow x>1$ $\Big\}\ x \geq 2$

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$g(x) = \sqrt{-x^2+4x-4} \rightarrow -x^2+4x-4 \geq 0 \rightarrow x^2-4x+4 \leq 0 \rightarrow (x-2)^2 \leq 0$

$\boxed{x=2}$

$D_{gof(x)} = \{x \,/\, \underbrace{x \in D_f}_{\mathrm{I}\ x\geq 2},\ \underbrace{f(x) \in D_g}\}$

$\sqrt{\log_2^{x-1}} = 2 \rightarrow \log_2^{x-1} = 4 \rightarrow x-1 = 16$

$\mathrm{II}\quad x = 17$

$\mathrm{I} \cap \mathrm{II} \rightarrow x = 17 \qquad D_{gof(x)} = \{17\}$

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$f(x) = 2x-5$

$g(x) = x^2-3x+8$

$fog(x) = gof(x) \rightarrow 2(x^2-3x+8)-5 = \overbrace{(2x-5)^2-3(2x-5)+8}^{4x^2+25-20x-6x+15+8}$

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$\rightarrow 2x^2-6x+16-5 = 4x^2-26x+48 \rightarrow 2x^2-20x+37 = 0$

$\alpha+\beta = \frac{-b}{a} = \frac{20}{2} = 10$

$\alpha\beta = \frac{c}{a} = \frac{37}{2}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 100-2\left(\frac{37}{2}\right) =$

$100-37 = 63$ جواب

۳-

$gof(x) = 5x^2+11$

$f(x) = 2x \rightarrow f(x)=t \rightarrow 2x=t \rightarrow x = \frac{t}{2}$

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$g(t) = 5\left(\frac{t}{2}\right)^2+11 = \frac{5t^2}{4}+11 \rightarrow g(x) = \frac{5x^2}{4}+11$

$g(x-7) = \frac{5(x-7)^2}{4}+11 = \frac{5(x^2-14x+49)}{4}+11 = \frac{5x^2-70x+289}{4}$

$\rightarrow \left|\begin{array}{l} \frac{-b}{2a} = \frac{70}{10} = 7 \\ \frac{5(7)^2-70(7)+289}{4} = \frac{245-490+289}{4} = 11 \end{array}\right.$ جواب

جواب

-۴

$f(x) = 3x - 4 \longrightarrow f(g(x)) = 3g(x) - 4$

$fog(x) = 3x^2 - 6x - 1$

$\longrightarrow 3g(x) - 4 = 3x^2 - 6x - 1 \longrightarrow 3g(x) = 3x^2 - 6x + 3 \longrightarrow$

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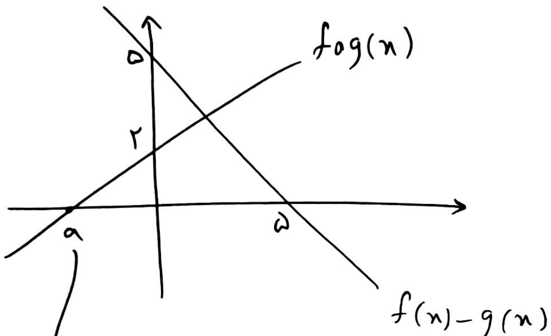
$g(x) = x^2 - 2x + 1 = (x-1)^2$

$gof(x) = (3x - 4 - 1)^2 = (3x-5)^2 = 9x^2 - 30x + 25$

جواب

-۵



$f(x) - g(x) = -x + 5$

$2g(x) = 3f(x) - 14$

$fog(x) = \frac{2}{a}x + 2$

$a < 0$

$\longrightarrow \begin{cases} f(x) - g(x) = 5 - x \\ 3f(x) - 2g(x) = 14 \end{cases} \xrightarrow{\times -2} \begin{cases} -2f(x) + \cancel{2g(x)} = 2x - 10 \\ 3f(x) - \cancel{2g(x)} = 14 \end{cases}$

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$\boxed{f(x) = 2x + 4}$

$g(x) = f(x) - 5 + x = \boxed{3x - 1}$

$fog = 2(3x - 1) + 4 = 6x + 2$

$6x + 2 = \frac{2}{a}x + 2 \longrightarrow \frac{2}{a} = 6 \longrightarrow a = \frac{1}{3} \xrightarrow{a<0} a = \frac{-1}{3}$

جواب

[illegible]

۶ـ

$f(x) = 9^x \qquad fog(x) \leqslant gof(x)$

$g(x) = \log_3^x$

$fog(x) = 9^{\log_3^x} = x^{\log_3^9} = x^2 \to fog(x) = x^2$

$gof(x) = \log_3^{9^x} = x \log_3^9 = 2x \to gof(x) = 2x$

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$\to x^2 \leqslant 2x \to x^2 - 2x \leqslant 0 \to x(x-2) \leqslant 0$

x		0		2	
	$+$		$-$		$+$

$\to [0,2] = \{0,1,2\} \to$ شامل ۳ عدد صحیح است.

۷ـ

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$

$f\left(\overbrace{x - \frac{2}{x}}^{t}\right) = \overbrace{\left(x - \frac{2}{x}\right)^2}^{t^2} + \overbrace{x - \frac{2}{x}}^{t} \longrightarrow f(x) = x^2 + x$

$\left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 4$

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۸ـ

$f(x) = \log_2^x \to x > 0 \qquad D_{fog} \to A \qquad R_{fog} \to B \qquad A \cap B = ?$

$g(x) = 8x - x^2 \to \mathbb{R}$

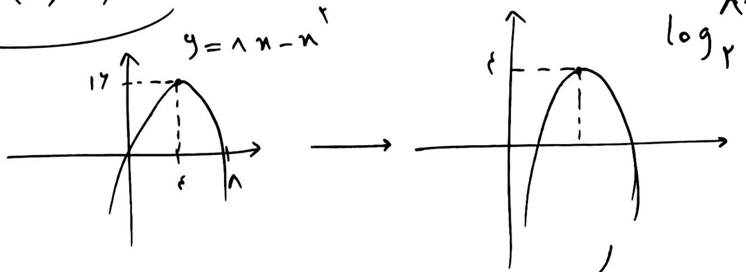
$D_{fog} : \{x \mid \underbrace{x \in D_g}_{\mathrm{I}\ \mathbb{R}},\ \underbrace{g(x) \in D_f}\}$

$8x - x^2 > 0 \to x^2 - 8x < 0$

x		0		8	
	$+$		$-$		$+$

$\to (0,8)\ \mathrm{II}$

$\mathrm{I} \cap \mathrm{II} \to (0,8) \longrightarrow A = (0,8)$



$fog(x) = \log_2^{8x - x^2}$

$D_{fog(x)} = (0,8)$

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$B = (-\infty, 4] \longleftarrow R_{fog(x)} = (-\infty, 4]$

$A \cap B = (0,4] = \{1,2,3,4\}$ جواب ← شامل ۴ عضو صحیح است

تغییر

۹-

$$f\circ f(x) = 1 - 3f^2(x)$$

$$f(x)=t \rightarrow f(t) = 1-3t^2 \rightarrow \boxed{f(x) = 1-3x^2}$$

$$\left.\begin{array}{l} g(x) = x-1 \\ h(x) = x^3+2x+1 \end{array}\right\} \quad h(g(x)) = (x-1)^3 + 2(x-1)+1 = x^3-1+3x-3x^2+2x-2+1$$

$$= x^3 - 3x^2 + 5x - 2$$

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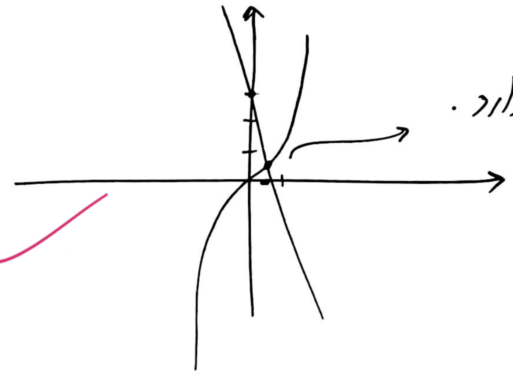
$$\longrightarrow h(g(x)) = f(x) \longrightarrow x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2}$$

$$\longrightarrow x^3 + 5x - 2 = 1 \longrightarrow x^3 = 3 - 5x$$



۱ ریشه دارد.

۱۰-

$$f(x) = x - \frac{4}{x}$$

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$$g\circ f(x) = ax^3 + 2x$$

$$g(3) = 8$$

$$\downarrow$$

$$f(x) = 3 \rightarrow x - \frac{4}{x} = 3 \rightarrow \frac{x^2-4}{x} = 3 \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0$$

$x = 4$ یا -1

$$\text{if } x = 4 \longrightarrow a(4)^3 + 2(4) = 8 \rightarrow 64a = 0 \rightarrow \boxed{a = 0}$$

$$\text{if } x = -1 \longrightarrow a(-1)^3 + 2(-1) = 8 \rightarrow -a = 10 \rightarrow \boxed{a = -10}$$

جواب $a = -10$ یا 0