تکلیف ۵ سوگند زارعان ۱۴۰۴/۷/۷

۱)

$D_{gof} = \{x \in D_f \mid f(x) \in D_g\} \Rightarrow$

$x \in D_f : \log_2^{x-1} \geq 0 \quad x-1 \geq 1 \Rightarrow x \geq 2$

$D_g : -x^2+4x-4 \geq 0 \quad x^2-4x+4 \leq 0 \quad (x-2)^2 \leq 0 \quad x=2$

$\sqrt{\log_2^{x-1}} = 2 \Rightarrow \log_2^{x-1} = 4 \Rightarrow x-1 = 16 \quad x = 17$

$D_{gof} = \{17\}$

۲)

$gof = (2x-5)^2 - 3(2x-5) + 8 = 4x^2+25-20x-6x+15+8$

$= 4x^2-26x+48$

$fog = 2(x^2-3x+8) - 5 = 2x^2-6x+16-5 = 2x^2-6x+11$

$2x^2-6x+11 = 4x^2-26x+48 \Rightarrow 2x^2-20x+37 = 0$

$\alpha^2+\beta^2 = S^2-2P \Rightarrow S = -\frac{(-20)}{2} = 10 \quad P = \frac{37}{2}$

$\alpha^2+\beta^2 = 100 - 2\times\frac{37}{2} = 100-37 = 63$

۳)

$gof = g(2x) = 5x^2+11 \Rightarrow 2x = t \quad x = \frac{t}{2}$

$g(t) = \frac{t^2}{4}\times 5+11 \Rightarrow g(x) = \frac{5x^2}{4}+11 \quad g(x-7) = \frac{5(x-7)^2}{4}+11$

کمترین مقدار این رابطه زمانی اتفاق می‌افتد که $x=7$ باشد. چون $\frac{5(x-7)^2}{4}$ همواره مثبت است و کمترین مقدار آن برابر صفر است

$\frac{5(7-7)^2}{4}+11 = 11$

۴)

$f(g(x)) = 3x^2-6x-1 \Rightarrow f(g(x)) = 3g-4 = 3x^2-6x-1$

$\Rightarrow \frac{3x^2-6x+3}{3} = g \Rightarrow g = x^2-2x+1 = (x-1)^2$

$gof = g(f(x)) = (3x-4-1)^2 = (3x-5)^2 = 9x^2+25-30x$

۵)

$fog(x) = \begin{matrix}(0,5)\\(5,0)\end{matrix} \Rightarrow m = -1 = \frac{5}{-5}$; $fog = \begin{matrix}(0,2)\\(a,0)\end{matrix}$ $\quad m = -\frac{2}{a}$

$fog(x) = -x+5$; $fog(x) = -\frac{2}{a}x+2$

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۵ -

$2g(x) = 3f(x) - 14 \Rightarrow 3f(x) - 2g(x) = 14$

$f - g = -x + 5$

$$\times -2 \left| \begin{array}{l} 3f(x) - 2g(x) = 14 \\ -2f + 2g = +2x - 10 \end{array} \right.$$

$f(x) = 2x + 4$

$\Rightarrow f(x) = 2x + 4$

$g(x) = 2x + 4 - g(x) = -x + 5 \Rightarrow g(x) = 3x - 1$

$fog = -\frac{2}{a} + 2 \qquad fog = 2(3x-1) + 4 = 6x - 2 + 4 = 6x + 2$

$-\frac{2}{a} = 6 \qquad 6a = -2 \Rightarrow \boxed{a = -\frac{1}{3}}$

۶

$fog = 9^{\log_3 x} = x^{\log_3 9} = x^2 \qquad gof = \log_3 9^x = x \log_3 9 = 2x$

$gof(x) \geq fog(x) \qquad 2x \geq x^2 \Rightarrow x^2 - 2x \leq 0$

$x = 0 \quad x = 2$ ریشه‌ها

$+ \quad 0 \quad - \quad 2 \quad +$

$\{0, 1, 2\}$

۷

$\frac{x^2 - 2}{x} = x - \frac{2}{x} \qquad f\left(x - \frac{2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \rightarrow x - \frac{2}{x}$

$\left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 4 \qquad \left(x - \frac{2}{x}\right)^2$

اگر داشته باشیم

$t = x - \frac{2}{x}$

$f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

۸

$fog(x) = \log_2^{8x - x^2} \qquad D_{fog} = 8x - x^2 > 0 \qquad x(8 - x) > 0$

$- \quad 0 \quad + \quad 8 \quad -$

$D_{fog} = (0, 8) \qquad R_{fog} = \log_2^{R_g}$

$R_g = 8x - x^2 : \frac{-8}{-2} = 4$

رأس سهمی $(4, 16)$

و برد تابع g برابر است با $(-\infty, 16]$

$R_{fog} = \log_2^{(-\infty, 16]} = (-\infty, 4]$

$(-\infty, 4] \cap (0, 8) = (0, 4]$

$0 \quad 4 \quad 8$

ولنز العمان

۹)

$f\circ f(x) = f(f(x)) = 1-3f^2(x) \Rightarrow f(x) = 1-3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1)+1 = x^3-3x^2+3x+2x+1-2$

$-2+x^3-3x^2+3x+2x+1 = f(x) = 1-3x^2 \Rightarrow x^3+5x-2=0$

۱۰)

$g(3) = 8 \Rightarrow x - \frac{4}{x} = 3 \xleftrightarrow{\times x} x^2-3x-4=0$

$x=4$ قق $x=-1$ قق

$\Rightarrow g(f(4)) = a(4)^3 + 2(4) = 8$

$64a + 8 = 8$

$64a = 0 \Rightarrow a = 0$

$g(f(-1)) \Rightarrow a(-1)^3 + 2(-1) = 8$

$-a - 2 = 8 \Rightarrow a = -10$