تکلیف ۵ — آرسام گندزاری — ۱۴۰۴/۷/۷

۱۹٫۵ آفرین!!

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(1)

$D_{gof}=\{x\in D_f \mid f(x)\in D_g\} \Rightarrow$

$x\in D_f: \log_2^{x-1} \geq 0,\quad x-1\geq 1 \Rightarrow x\geq 2$

$D_g: -x^2+4x-4\geq 0,\quad x^2-4x+4\leq 0,\quad (x-2)^2\leq 0,\quad x=2$

$\sqrt{\log_2^{x-1}}=2 \Rightarrow \log_2^{x-1}=4 \Rightarrow x-1=16,\quad x=17$

$D_{gof}=\{17\}$

2

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(2)

$gof=(2x-5)^2-3(2x-5)+8=4x^2+25-20x-6x+15+8$

$=4x^2-26x+48$

$fog=2(x^2-3x+8)-5=2x^2-6x+16-5=2x^2-6x+11$

2

$2x^2-6x+11=4x^2-26x+48 \Rightarrow 2x^2-20x+37=0$

$\alpha^2+\beta^2=S^2-2P \Rightarrow S=-\frac{(-20)}{2}=10,\quad P=\frac{37}{2}$

$\alpha^2+\beta^2=100-2\times\frac{37}{2}=100-37=63$

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(3)

$gof=g(2x)=5x^2+11 \Rightarrow 2x=t,\quad x=\frac{t}{2}$

$g(t)=\frac{t^2}{4}\times 5+11 \Rightarrow g(x)=\frac{5x^2}{4}+11 \qquad g(x-7)=\frac{5(x-7)^2}{4}+11$

2

کمترین مقدار این رابطه زمانی اتفاق می‌افتد که $x=7$ باشد. چون $\frac{5(x-7)^2}{4}$ همواره مثبت است و کمترین مقدار آن صفر است

$\frac{5(7-7)^2}{4}+11=11$]

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(4)

$f(g(x))=3x^2-6x-1 \Rightarrow f(g(x))=3g-4=3x^2-6x-1$

$\Rightarrow \frac{3x^2-6x+3}{3}=g \Rightarrow g=x^2-2x+1=(x-1)^2$

2

$gof=g(f(x))=(3x-4-1)^2=(3x-5)^2=9x^2+25-30x$

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(5)

$f(x)=$ $(0,5)$, $(5,0)$ $\Rightarrow m=-1=\frac{5}{-5}$ | $fog=$ $(0,2)$, $(9,0)$

$-x+5$ | $m=-\frac{2}{9}$

$fog(x)=-\frac{2}{9}x+2$

سهیل زارعیان

۵)

$2g(x) = 3f(x) - 14 \Rightarrow 3f(x) - 2g(x) = 14$

$f - g = -x + 5$

$$\begin{cases} 3f(x) - 2g(x) = 14 \\ -2f + 2g = \cancel{+2x-10} \; -x+5 \end{cases} \quad \times -2$$

$f(x) = 2x + 4$

۲

$\Rightarrow f(x) = 2x + 4$

$g(x) = 2x + 4 - g(x) = -x + 5 \Rightarrow g(x) = 3x - 1$

$fog = -\frac{2}{a} + 2 \qquad fog = 2(3x - 1) + 4 = 6x - 2 + 4 = 6x + 2$

$-\frac{2}{a} = 6 \qquad 6a = -2 \Rightarrow$ ~~[illegible]~~ $a = -\frac{1}{3}$

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۶)

$fog = 9^{\log_3 x} = x^{\log_3 9} = x^2 \qquad gof = \log_3 9^x = x \log_3 9 = 2x$

۲

$gof(x) \geq fog(x) \qquad 2x \geq x^2 \Rightarrow x^2 - 2x \leq 0$

$x = 0 \quad x = 2$ ریشه‌ها

$+ \;\; 0 \;\; - \;\; 2 \;\; +$

$\{0, 1, 2\}$

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۷)

$\frac{x^2 - 2}{x} = x - \frac{2}{x} \qquad f\left(x - \frac{2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \quad x - \frac{2}{x}$

$\left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 4 \qquad \left(x - \frac{2}{x}\right)^2$

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آن را [illegible] باشیم

$t = x - \frac{2}{x}$

$f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

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۸)

$fog(x) = \log_2^{8x - x^2} \qquad D_{fog} = 8x - x^2 > 0 \qquad x(8 - x) > 0$

$- \;\; 0 \;\; + \;\; 8 \;\; -$

$D_{fog} = (0, 8) \qquad R_{fog} = \log_2^{R_g}$

۲

$R_g = 8x - x^2 : \frac{-8}{-2} = 4$ راس سهمی $(4, 16)$

$(-\infty, 16]$ [illegible] برابر است با

$R_{fog} = \log_2^{(-\infty, 16]} = (-\infty, 4]$

$(-\infty, 4] \cap (0, 8) = (0, 4]$

$0 \quad 4 \quad 8$

سولنز العال

۹) ۱٫۵

$fof(x) = f(f(x)) = 1 - \cancel{3}f^2(x) \Rightarrow f(x) = 1 - 3x^2$

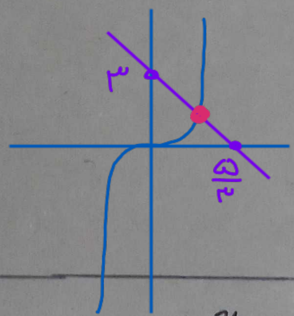
$h(g(x)) = (x-1)^3 + \frac{2x+1}{(x-1)} = x^3 - 3x^2 + 3x + 2x + 1 - 2$

$-2 + x^3 - 3x^2 + 3x + 2x + 1 = f(x) = 1 - 3x^2 \Rightarrow x^3 + 5x - 2 = 0$

$x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2}$

$x^3 = -5x + 3$

یک ریشه دارد



۱۰) ۲

$g(3) = 8 \Rightarrow x - \frac{4}{x} = 3 \xleftrightarrow{\times x} x^2 - 3x - 4 = 0$

$x = 4$ $x = -1$ $\Rightarrow g(f(4)) = a(4)^3 + 2(4) = 8$

ق ق ق ق

$64a + 8 = 8$

$64a = 0 \Rightarrow a = 0$

$g(f(-1)) \Rightarrow a(-1)^3 + 2(-1) = 8$

$-a - 2 = 8 \Rightarrow a = -10$