تکلیف شماره ۸ — سارینا ستاری

①

$D_{g\circ f} = \{x \mid x \in D_f,\ f(x) \in D_g\}$

$\log_2^{x-1} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2$

$x-1>0 \Rightarrow x>1$

$\Rightarrow x \in [2, +\infty)$ *

$-x^2+4x-4 \geq 0 \Rightarrow x^2-4x+4 \leq 0 \Rightarrow (x-2)^2 \leq 0 \Rightarrow x=2$

$\Rightarrow \sqrt{\log_2^{x-1}} = 2 \Rightarrow x-1=16 \Rightarrow x=17$ □

$\boxed{*, \square \Rightarrow D_{g\circ f} = \{17\}}$

②

$f\circ g = 2x^2-6x+16-5 = 2x^2-6x+11$

$g\circ f = 4x^2+20-20x^2-6x+15+18 = 4x^2-6x+28$

$\Rightarrow 4x^2-6x+28 = 2x^2-6x+11$

$\Rightarrow 2x^2-20x+17=0$

$\alpha^2+\beta^2 = S^2-2P = 100-17 = 83$

③

$2x = t \Rightarrow x = \frac{t}{2}$

$g(t) = \frac{5t^2}{4}+11 \Rightarrow g(x) = \frac{5}{4}x^2+11 \Rightarrow g(x-7) = \frac{5}{4}x^2+\frac{245}{4}+\frac{70x}{4}+11$

$= \frac{5}{4}x^2+\frac{70x}{4}+\frac{289}{4}$

$x_s = \frac{70}{4}\times\frac{2}{5} = 7 \Rightarrow y_s = \frac{-245}{4}$

④

$3g-4 = 3x^2-6x-1$

$g = \underbrace{x^2-2x+1}_{(x-1)^2} \Rightarrow$ ~~$g\circ f = 9x^2+16-24x-6x+8+1=9x^2-30x+25$~~

$g(3x-4) = (3x-5)^2 = 9x^2+25-30x$

⑤

$g(x) = ax+b$

$f(x) = cx+d$

$2ax+2b = 3cx+3d-14$

$\Rightarrow 3c = 2a \overset{*}{\Rightarrow} 3a-3 = 2a \Rightarrow a=3 \Rightarrow c=2$

$2b = 3d-14 \overset{*}{\Rightarrow} 2b = 3b+1 \Rightarrow b=-1 \Rightarrow d=4 \Big\} \Rightarrow$

$g(x) = 3x-1$

$f(x) = 2x+4$

$\hookrightarrow f\circ g = 6x+2$

$f-g = (c-a)x+d-b$ ~~$= cx-ax+d-b$~~

$= -x+5$

$\Rightarrow c-a=-1 \Rightarrow c=a-1$

$d-b=5 \Rightarrow d=b+5 \Big\}$ *

$\Rightarrow 6x+2=0 \Rightarrow x=-\frac{1}{3}$

$\Rightarrow a=-\frac{1}{3}$

⑥

$fog = 9^{\log_3^x} = x^{\log_3^9} = x^2$

$gof = \log_3^{9^x} = x\log_3^9 = 2x$

$x^2 \le 2x \Rightarrow x^2 - 2x \le 0$

$\begin{array}{c|c|c} & 0 \quad 2 & \\ + & - & + \end{array} \Rightarrow x \in [0,2]$

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⑦

$f\left(x - \frac{2}{x}\right) = \underbrace{x^2 + \frac{4}{x^2} - 4}_{=\left(x-\frac{2}{x}\right)^2} + \underline{x - \frac{2}{x}} \Rightarrow f(x) = x^2 + x$

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⑧

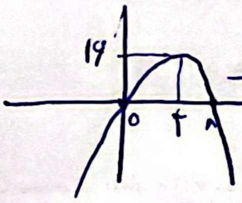
$D_{fog} = \{x \mid x \in D_g , g(x) \in D_f\}$ $\quad x \in D_g = R$ *

$D_f = x > 0 \Rightarrow 8x - x^2 > 0$

$fog = \log_2^{8x - x^2}$

$\begin{array}{c|c|c} & 0 \quad 8 & \\ - & + & - \end{array} \Rightarrow x \in (0,8)$ □

$* \cap □ \Rightarrow x \in (0,8) = A$



$\to R = (-\infty, 16]$

$\xrightarrow{0<x<8} R = (0,16] \Rightarrow \log_2^{(0,16]} = (1,4] = R_{fog} = B$

$A \cap B = \{1,2,3\}$

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⑨

$f(x) = t \Rightarrow f(t) = 1 - 3t^2 \Rightarrow f(x) = 1 - 3x^2$

$h(g(x)) = x^3 + 3x - 3x^2 - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 5x - 2$

$= 1 - 3x^2$

$\Rightarrow x^3 + 5x - 3 = 0 \longrightarrow$ ۱ ریشه دارد

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⑩

$g\left(x - \frac{4}{x}\right) = ax^3 + 2x$

$g(3) = 8$

$\Rightarrow x - \frac{4}{x} = 3 \Rightarrow x^2 - 4 = 3x \Rightarrow x^2 - 3x - 4 = 0$

$(x-4)(x+1) \to x = -1 \;,\; x = 4$

$x = -1 \Rightarrow -a + (-2) = 8 \Rightarrow a = -10$

$x = 4 \Rightarrow 64a + 8 = 8 \Rightarrow a = 0$