مهشید جدیدی

Subject: « تکلیف شماره ۸ »

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①

$f(x)=\sqrt{\log_2^{(x-1)}}$ $\qquad g(x)=\sqrt{-x^2+4x-4}$

$D_{gof}=\{x\in D_f \,;\, f(x)\in D_g\}$ → تعریف دامنه ترکیب تابع

$\log_2^{(x-1)}$ { $x-1>0 \to x>1$ ، $\log_2^{(x-1)}\ge 0 \to (x-1)\ge 1 \to x\ge 2$ } $\to D_f=[2,+\infty)$ ①

شرط دوم → $\sqrt{\log_2^{(x-1)}}\in D_g$ $\qquad D_g = -(x^2-4x+4)\ge 0$

به ازای هیچ ← $-(x-2)^2\ge 0$

$D_{g(x)}=\{2\}$ مقدار عبارت منفیست به غیر از $x=2$ که صفر میشه

$\sqrt{\log_2^{(x-1)}}=2 \to \log_2^{(x-1)}=4 \to (x-1)=16 \to x=17$ ②

$① \cap ② \Rightarrow D_{gof}=\{17\}$

②

$f(x)=2x-5$ $\qquad g(x)=x^2-3x+8$

$fog(x)=gof(x) \longrightarrow 2(x^2-3x+8)-5=(2x-5)^2-3(2x-5)+8$

$2x^2-\cancel{6x}+11=4x^2-20x+25-\cancel{6x}+3 \longrightarrow 2x^2-20x+37=0$

مجموع مربعات $=\alpha^2+\beta^2=S^2-2P \longrightarrow S=\frac{-b}{a}=\frac{+20}{2}=10$ ، $P=\frac{c}{a}=\frac{37}{2}$

$S^2-2P=100-\cancel{2}\frac{37}{\cancel{2}}=63$

Arman

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$gof(x) = 5x^2+11 \qquad f(x)=2x \qquad \min g(x-7)$

$f(x)=2x \longrightarrow 2x=t \to x=\frac{t}{2} \qquad f(x)=t$

$g(t)=5\left(\frac{t}{2}\right)^2+11 \longrightarrow g(x)=\frac{5}{4}x^2+11$

$g(x-7)=\frac{5}{4}(x-7)^2+11 \longrightarrow \frac{5}{4}(x^2-14x+49)+11 = \frac{5}{4}x^2-\frac{35}{2}x+\frac{5}{4}(49)+\frac{11}{1}$

$245+44$

$g(x-7)=\frac{5}{4}x^2-\frac{35}{2}x+\frac{289}{4} \leadsto x=\frac{+\frac{35}{2}}{\frac{10}{4}}=\frac{\cancel{4}\times 35}{\cancel{2}\times 5\times \cancel{2}}=7$

$\min g(x)=\frac{5}{4}\times 49-\frac{35}{2}\times 7+\frac{289}{4}=\frac{245}{4}-\frac{490}{4}=\frac{-245}{4}+\frac{289}{4}$

$= \min g(x-7) = 11$

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$f(x)=3x-4 \qquad fog(x)=3x^2-6x-1 \qquad gof(x)=?$

$f(g(x))=3g-4=3x^2-6x-1 \leadsto 3g(x)=3x^2-9x+3$

$g(x)=x^2-2x+1 \longrightarrow g(x)=(x-1)^2$

$g(f(x))=(3x-4-1)^2=9x^2-30x+25$

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جمشید حبیبی

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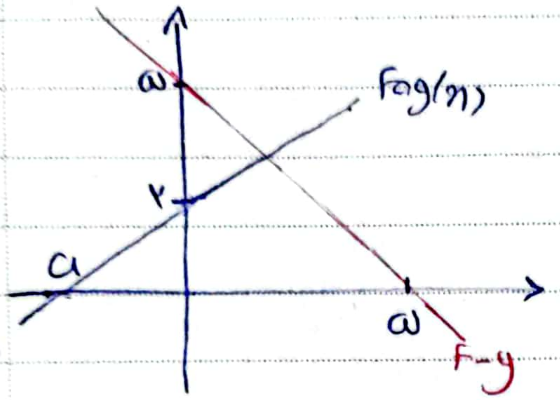

$2g(x) = 3f(x) - 14$

$$\begin{cases} -2\,f(x) - g(x) = -x + 5 \quad (\times -2 \Rightarrow -2x + 10) \\ 3f(x) - 2g(x) = 14 \end{cases}$$

$$\begin{cases} 2f(x) - 2g(x) = -2x + 10 \\ 3f(x) - 2g(x) = 14 \end{cases} \rightarrow f(x) = 14 + 2x - 10 \Rightarrow f(x) = 2x + 4$$

$\rightarrow 2x + 4 + x - 5 = g(x) \rightarrow g(x) = 3x - 1$

$fog(x) = 2(3x - 1) + 4 \rightarrow 6x + 2 = 0 \rightarrow 6x = -2 \rightarrow x = -\frac{1}{3} = a$

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$f(x) = 9^x \qquad g(x) = \log_3^x \qquad fog(x) \le gof(x)$

$x > 0$

$fog(x) = 9^{\log_3^x} = x^{\log_3^9} = x^2$

$gof(x) = \log_3^{9^x} = x \log_3^9 = 2x$

$\rightarrow x^2 \le 2x \rightarrow x^2 - 2x \le 0$

$x(x-2) \le 0$

$\begin{array}{c|c|c|c|c} & 0 & & 2 & \\ \hline + & 0 & - & 0 & + \end{array} \rightarrow (0, 2]$

دامنه صفر نمی‌گیرد

تعداد اعداد صحیح $= \{1, 2\}$ — ۲ عدد صحیح

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$x - \frac{2}{x} =$ درون پرانتز

$f\left(\frac{x^2 - 2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \qquad f(x) = ?$

$\rightarrow \left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 2(x)\left(\frac{2}{x}\right) \quad -4$

$f\left(x - \frac{2}{x}\right) = \left(x - \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right) \rightsquigarrow f(x) = x^2 + x$

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$F(x)=\log_2^x \qquad g(x)=8x-x^2$

$A=D_{fog} \quad B=R_{fog} \Rightarrow A\cap B=?$

$D_{fog(x)}=\{\underbrace{x\in D_g}_{\mathbb{R}} ; g(x)\in D_f\}$

$\begin{array}{ccccc} & 0 & & 8 & \\ - & | & + & | & - \end{array}$

$D_g=\mathbb{R}$ $\quad \to 8x-x^2>0 \quad x(8-x)>0 \to (0,8)$

$D_{f(x)}= x>0 \qquad D_{fog(x)}=(0,8)=A$

برای $-x^2+8x$

$fog(x)=\log_2^{8x-x^2} \xrightarrow{a<0} \begin{cases} x_{min}=\frac{-b}{2a}=\frac{-8}{-2}=4 \\ y_{max}=-16+32=16 \to \log_2^{16}=4 \end{cases}$

$A\cap B=(0,4]$ $\qquad R_{gof}=(-\infty,4]$

$\to$ عضو صحیح $\{1,2,3,4\}$

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$fof(x)=1-3f^2(x) \qquad g(x)=x-1 \qquad h(x)=x^3+2x+1$

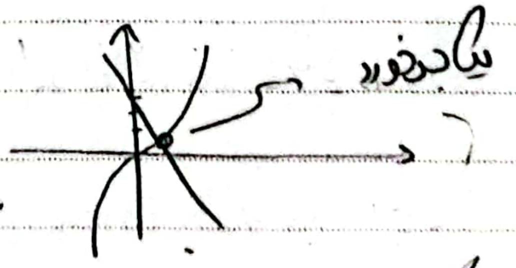
$h(g(x))=(x-1)^3+2(x-1)+1 \qquad h(g(x))=f(x)$

$f(f(x))=1-3(f(x))^2 \Rightarrow f(x)=1-3x^2$

$\to x^3-1-3\overset{+3x}{(x+1-1)}(x-1) = x^3 \cancel{-1}+3x^2-\cancel{3x}+\cancel{2x}-2\cancel{+1}$

$\cancel{+3x^2}-3x$

$\to x^3+5x-3=0 \to x^3=-5x+3$



پس معادله یک جواب دارد

Arman

عشر بیجری

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$f(x) = x - \frac{4}{x}$ $\quad g \circ f(x) = ax^3 + 2x$ $\quad g(3) = 8$ $\quad a = ?$

$g(3) = 8 \longrightarrow f(x) = 3 \longrightarrow x - \frac{4}{x} = 3$ $\quad \frac{x^2 - 4}{x} = \frac{3}{1}$

$x^2 - 4 = 3x \rightarrow x^2 - 3x - 4 = 0 \rightarrow x = -1$ $\quad ax^3 + 2x = 8 \rightarrow g(3) = 8$

$(x+1)(x-4) \rightarrow x = 4$

$x = -1 \longrightarrow -a - 2 = 8 \longrightarrow a = -10$

$x = 4 \longrightarrow 64a + 8 = 8 \longrightarrow a = 0$