!! آفرین ۲۰ دوازدهم تجربی B مهرنیا ریاضی

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$g(x)=\sqrt{-x^2+4x-4} \qquad f(x)=\sqrt{\log_2^{(x-1)}} \qquad D_{gof(x)} = ?$

همواره منفی پس زیر رادیکال باید صفر شود

$g(x)=\sqrt{-(x^2-4x+4)} = \sqrt{-(x-2)^2} \longrightarrow -(x-2)^2 \geq 0 \longrightarrow x-2=0 \quad x=2$

$g(f(x)) = \sqrt{-(\sqrt{\log_2^{(x-1)}}-2)^2} \longrightarrow \sqrt{\log_2^{(x-1)}} = 2 \quad x-1=16 \quad x=17$

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$f(x)=2x-5 \qquad g(x)=x^2-3x+8$

$fog(x)=gof(x) \rightarrow f(g(x)) = g(f(x)) \rightarrow 4x^2-26x+48 = 2x^2-6x+11$

$2x^2-20x+37=0 \qquad \alpha^2+\beta^2 = S^2-2P \qquad S=\frac{-b}{a}=10 \quad P=\frac{c}{a}=\frac{37}{2}$

$\alpha^2+\beta^2 = 100-2(\frac{37}{2}) = 100-37 = 63$

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۳

$gof(x) = 5x^2+11 \qquad f(x)=2x \qquad g(x-7) = \frac{5(x-7)^2}{4}+11$

$f: \quad 2x=t \quad x=\frac{t}{2} \qquad g(\frac{t}{2}) = \frac{5t^2}{4}+11 \rightarrow g(x) = \frac{5x^2}{4}+11 \qquad g(x-7) = \frac{5(x-7)^2}{4}+11$

$\xrightarrow[x=7]{min} \frac{5(7-7)^2}{4}+11 = 11$

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$f(x)=3x-4 \qquad fog(x) = 3x^2-9x-1 \qquad gof(x) = ?$

$g(x) = x^2-2x+1 \qquad g(f(x)) = (3x-4-1)^2 = (3x-5)^2 = 9x^2-30x+25$

$g(x) = (x-1)^2$

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۵

$\begin{cases} f(x)-g(x) = -x+5 \\ 2g(x)-3f(x) = -14 \end{cases} \longrightarrow \begin{matrix} f(x)=2x+4 \\ g(x)=3x-1 \end{matrix} \qquad fog(x) = f(g(x)) = 6x+2$

$6(a)+2=0 \qquad 6a=-2 \qquad a=-\frac{2}{6} = -\frac{1}{3}$

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$f(x) = 9^x \qquad g(x) = \log_3 x \qquad f\circ g(x) \leq g\circ f(x) \qquad \log_3 x \to x>0 \quad \mathrm{I}$

$f(g(x)) = 9^{\log_3 x} = x^{\log_3 9} = x^2 \;/\; g(f(x)) = 2x = \log_3 9^x \qquad \mathrm{I}\cap\mathrm{II}$ ۲

$x^2 \leq 2x \qquad x^2 - 2x \leq 0 \qquad x(x-2) \leq 0$ $\quad \begin{array}{ccccc} & 0 & & 2 & \\ + & & - & & + \end{array}$ II $\quad (0,2]$

۲ عبارت صحیح

۷

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$

$f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x} = \left(x - \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right)$ ۲

$f(t) = t^2 + t \longrightarrow f(x) = x^2 + x$

۸

$f(x) = \log_2 x \qquad g(x) = 8x - x^2 \qquad D_{fog} = A \qquad R_{fog} = B$

$\begin{array}{ccccc} & 0 & & 8 & \\ + & & - & & + \end{array}$

$fog(x) = f(g(x)) = \log_2 (8x - x^2) \qquad 8x - x^2 > 0 \qquad x^2 - 8x < 0, \; x(x-8) < 0 \qquad D_{fog} = (0,8) = A$ ۲

$\frac{-b}{2a} = \frac{-8}{-2} = 4 \xrightarrow{x=4} \log_2^{32-16} = 4$

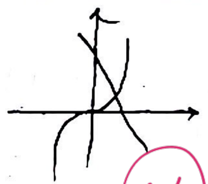
$-x^2 + 8x = -(x-4)^2 + 16 \qquad 0 < x < 8 \qquad -4 < x-4 < 4 \qquad 0 \leq (x-4)^2 < 16$

$-16 < -(x-4)^2 \leq 0 \qquad 0 < -(x-4)^2 + 16 \leq 16$

$R \;\; (0,16] \xrightarrow{\log_2} (0,4] = B \qquad A\cap B = (0,4] \cap (0,8) = (0,4] \to$ ۴ عبارت صحیح

۹

$f\circ f(x) = 1 - 3f^2(x) \qquad g(x) = x - 1 \qquad h(x) = x^3 + 2x + 1$



$f(x) = 1 - 3x^2 \qquad h(g(x)) = f(x) \to x^3 - 3x^2 + 5x - 2 = 1 - 3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x + 2x - 2$ $\qquad x^3 + 5x - 3 = 0$ ۲

یک ریشه دارد $\qquad x^3 = 3 - 5x$

۱۰

$f(x) = x - \frac{4}{x} \qquad g\circ f(x) = ax^3 + 2x \qquad g(3) = 8$

$f(4) = 4 - \frac{4}{4} = 3 \qquad g(f(4)) = g(3) = 8 \Rightarrow 64a + 8 = 8 \qquad 64a = 0 \qquad a = 0$

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$f(x) = 3 \qquad \frac{x^2-4}{x} = 3 \qquad (x-4)(x+1) = 0 \to x = 4, \; x = -1$

$g(f(-1)) = 8 = ax^3 + 2x \qquad -a - 2 = 8 \qquad a = -10$