مینا لطفی ۱۲ ریاضی سه‌شنبه

$D_{gof} = \{\underbrace{x \in D_f}\ |\ f(x) \in D_g\}$

$x-1>0 \to x>1$ (1)

$\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$ (2) $\to 1 \cap 2 \to x \geq 2$

$D_g \to -(x^2-4x+4) \geq 0 \to (x-2)^2 \leq 0 \to x=2 \to \sqrt{\log_2^{x-1}} = 2$

$\to \log_2^{x-1} = 4 \to \boxed{x=17}$

-2

$f(g(x)) = 2(x^2-3x+8)-5 = 2x^2-6x+11$

$g(f(x)) = (2x-5)^2-3(2x-5)+8 = 4x^2-20x+25-6x+23 =$

$4x^2-26x+48$

$\to 4x^2-26x+48 = 2x^2-6x+11$

$2x^2-20x+37=0 \to \alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 100-2\times\frac{37}{2} = 63$

-3

$g(2x) = 5x^2+11 \qquad 2x=t \to x=\frac{t}{2} \qquad g(t) = 5\times\frac{t^2}{4}+11$

$g(x) = 5\frac{x^2}{4}+11 \qquad g(x-7) = 5\frac{(x-7)^2}{4}+11 = 5\frac{(x^2-14x+49)}{4}+11$

$= \frac{5x^2}{4} - \frac{35}{2}x + \frac{289}{4}$

$x_S = \frac{35}{2}\times\frac{2}{5} = 7$

$y(S) = \frac{49\times5}{4} \quad \frac{245}{2} + \frac{289}{4} = +11$

-4

$f(g(x)) = 3g(x)-4 = 3x^2-6x-1 \to 3g(x) = 3x^2-6x+3$

$g(x) = x^2-2x+1 = (x-1)^2$

$g(3x-4) = (3x-5)^2 = 9x^2-30x+25$

-5

$f-g \Rightarrow y=-x+5 \qquad fog(x)=$

$(5,0)(0,5)$

$f(x)-g(x) = -x+5 \to 2x+4-g(x) = -x+5$

$\begin{cases} 3f(x)-2g(x) = 14 \\ -2f(x)+2g(x) = 2x-10 \end{cases} \to \begin{cases} f(x) = 2x+4 \\ g(x) = 3x-1 \end{cases}$

$f(g(x)) = f(3x-1) = 2(3x-1)+4 = 6x+2 \to 6x+2=0$

$x = -\frac{1}{3} = a$

۶-

$f(g(x)) \leq g(f(x)) \rightarrow$

$f(g(x)) = f(\log_3 x) = 9^{\log_3 x} = x^2$

$g(f(x)) = g(9^x) = \log_3 9^x = 2x$

$\rightarrow x^2 \leq 2x \rightarrow 0 \leq x \leq 2$

↓

۳ عدد صحیح

۷-

$f(\underbrace{x - \frac{2}{x}}_{t}) = (x - \frac{2}{x})^2 + x - \frac{2}{x} \qquad f(x) = x^2 + x$

۸-

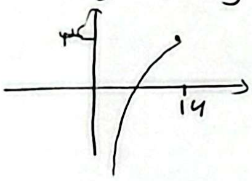
$D_{fog} = \{x \in \underbrace{D_g}_{R} \mid \underbrace{g(x) \in D_f}\} = (0,8)$ ①

$8x - x^2 > 0 \rightarrow x(8-x) > 0 \rightarrow 0 < x < 8$

$f(g(x)) = f(8x - x^2) = \log_2 (8x - x^2)$

$8x - x^2 \xrightarrow[0<x<8]{} 0 < 8x - x^2 \leq 16$

$\rightarrow -\infty < \log_2 (8x - x^2) \leq 4$ ②



$① \cap ② = (0,4] \rightarrow 1, 2, 3, 4$

۹-

$f(\underbrace{f(x)}_{t}) = 1 - 3f^2(x) = 1 - 3t^2 \rightarrow f(x) = 1 - 3x^2$

$h(x-1) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x \cancel{-1} + 2x - 2 \cancel{+1} =$

$x^3 - 3x^2 + 5x - 2$

$x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \rightarrow x^3 + 5x - 3 = 0$

۱۰-

$g(f(x)) = g(x - \frac{4}{x}) = ax^3 + 2x$

$\rightarrow x - \frac{4}{x} = 3 \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \rightarrow x = 4$

$\searrow x = -1$

$x = 4 \rightarrow g(3) = 64a + 8 = 8 \rightarrow a = 0$

$x = -1 \rightarrow g(3) = -a - 2 = 8 \rightarrow a = -10$