① $x \in D_f$: $x-1>0 \to x>1$, $\log_2^{(x-1)} \geq 0 \to x-1 \geq 1 \to x \geq 2 \to (x \geq 2)$

$f(x) \in D_g$: $g(x)=\sqrt{f(x^2-4x+4)}=\sqrt{-(x-2)^2} \to x=2, D_g=\{2\} \to f(x)=2$

$(x=17) \leftarrow 16=x-1 \leftarrow 4=\log_2^{x-1} \xleftarrow{\text{به توان ۲}} 2=\sqrt{\log_2(x-1)}$

$D_{g\circ f}=\{17\}$

---

② $f(g(x))=g(f(x))$: $\longrightarrow 2x^2-6x+11=4x^2-26x+48$

$g(x)=x^2-3x+8$, $f(x)=2x-5$

$2x^2-6x+16-5$; $4x^2-20x+25-6x+15+8$

$2x^2-6x+11$; $4x^2-26x+48$

$0=2x^2-20x+37$, $\alpha+\beta=\frac{20}{2}=10$, $\alpha.\beta=\frac{37}{2}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=100-2\times\frac{37}{2}=100-37=(63)$

---

③ $g(2x)=g(f(x))=5x^2+11$

$2x=t \to x=\frac{t}{2} \longrightarrow 5x^2+11=5\frac{t^2}{4}+11$

$g(t)=5\frac{t^2}{4}+11 \to g(x-7)=5\frac{(x-7)^2}{4}+11=\frac{5(x^2+49-14x)}{4}+11$

$\frac{5}{4}x^2-\frac{35}{2}x+\frac{289}{4}=g(x-7)$

$\frac{35/2}{5/2}=7=-\frac{b}{2a}$

$\frac{5}{4}\times 49-\frac{35}{2}\times 7+\frac{289}{4}=11 \to$ کمترین مقدار

---

④ $f(g(x)) \to 3g(x)-4=3x^2-6x-1 \to 3(g(x))=3x^2-6x+3$

$\to g(x)=x^2-2x+1$

$g(x)=(x-1)^2$

$g(f(x))=(3x-4-1)^2=9x^2-30x+25$

---

⑤ $f(g(x))=2g(x)+4 \to \begin{vmatrix}0\\2\end{vmatrix} \begin{vmatrix}a\\0\end{vmatrix} \to \frac{2}{-a}=2 \to (a=-1)$

$(f-g)(x)=-x+5$

$14=3f(x)-2g(x) \to 2(f(x)-g(x))+f(x)=14=-2x+10-14=-f(x)$

$-2x-4=-f(x) \to f(x)=2x+4$

---

⑥ $9^{\log_3^x} \leq \log_3^{9^x}$ ، دامنه $\log_3^x \to x \neq 0$ ، $x>0$ I

$x^{\log_3^9} \leq x\log_3^9 \to x^2 \leq 2x \to x^2-2x \leq 0 \to x \in [0,2]$ II

$\text{I} \cap \text{II} \to (0,2] \to \{2,1\} \to (1,2)$

---

⑦ $f(x-\frac{2}{x})=(x-\frac{2}{x})^2+(x-\frac{2}{x})$

$f(x)=x^2+x$

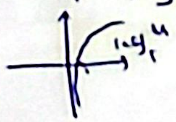
---

⑧ $f(g(x)) \to D_{f\circ g}$: $x \in D_g$, $g(x) \in D_f \to 8x-x^2>0$ $(0,8)$ (A)

$\to R_{f\circ g} \to \log_2^{8x-x^2}$ $(-\infty,\log_2^{16}] \to (-\infty,4]$ (B) $A\cap B=(0,4]$

$\max(8x-x^2)=\left|\frac{-8}{-2}=4 \atop 16\right.$

$x \neq 8,0$



دارای ۴،۳،۲،۱ ← ۴ عدد صحیح

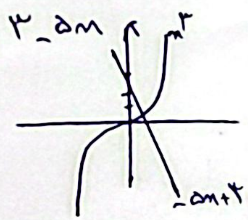
---

⑨ $f\circ f(x)=f(f(x)) \xrightarrow{f(x)=t} f(t)=1-3t^2 \to f(x)=1-3x^2$

$h(g(x))=(x-1)^3+2(x-1)+1=x^3-3x^2+3x-1+2x-2+1=x^3-3x^2+5x-2$

$x^3-3x^2+5x-2=-3x^2+1 \to x^3+5x-3=0 \to x^3=3-5x$

← یک ریشه دارد



---

⑩ $f(x)=3 \to \frac{x^2-4}{x}=3 \to x^2-4=3x \to x^2-3x-4=0$ $(x-4)(x+1)=0$ → (4) (-1)

$g(f(4))=8=ax^3+2x \to 8=64a+8 \to (a=0)$

$g(f(-1))=8=ax^3+2x \to 8=-a-2 \to (a=-10)$

[illegible] ۱۴ آذر B