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13 دوازدهم دختر

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۱) $\bullet f(x)=\sqrt{\log_2^{(x-1)}} \xrightarrow{D_f} \log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2 \to D_f:[2,+\infty)$

$R_f=[0,+\infty)$

$\bullet g(x)=\sqrt{-x^2+4x-4} \xrightarrow{D_g} -x^2+4x-4 \geq 0 \to -(x-2)^2 \geq 0 \to D_g=\{2\}$

$\bullet D_{gof}:\{x \in D_f \,,\, R_f \in D_g\} \to \{x \in [2,+\infty) \,,\, [0,+\infty) \in \{2\}\}$

$\xRightarrow{x-1=16} D_{gof}:\{17\}$

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۲) $fog(x)=gof(x) \Rightarrow 2(x^2-3x+8)-5=(2x-5)^2-3(2x-5)+8$

$\Rightarrow 2x^2-6x+16-5=4x^2+25-20x-6x+15+8$

$\Rightarrow 2x^2-20x+37=0 \longrightarrow S=10 \,,\, P=\frac{37}{2}$

$\alpha^2+\beta^2=S^2-2P=100-37=\boxed{63}$

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۳) $gof(x)=5x^2+11 \,,\, f(x)=2x$

$\Rightarrow g(2x)=5x^2+11 \longrightarrow g(x)=5(2x)^2+11$

$\Rightarrow g(x)=20x^2+11 \longrightarrow g(x-7)=20\underbrace{(x-7)^2}_{\text{برابر کمترین مقدار }0}+11$

$\boxed{\min_{g(x-7)}=11}$

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۴) $f(x)=3x-4 \,,\, fog(x)=3x^2-6x-1$

$\Rightarrow f(g(x))=3(g(x))-4=3x^2-6x-1 \Rightarrow g(x)=x^2-2x+1$

$\xrightarrow{gof(x)} (3x-4)^2-2(3x-4)+1 \Rightarrow \boxed{gof(x)=9x^2-30x+25}$

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۵) $(f-g)(x)=-x+5 \,,\, 14=2(\underbrace{f(x)-g(x)}_{(f-g)(x)})+f(x)$

$\Rightarrow f(x)=2x+4 \,,\, 14=3f(x)-2g(x) \longrightarrow g(x)=3x-1$

$fog(x)=\frac{-2}{a}x+2=2(3x-1)+4 \Rightarrow \boxed{a=\frac{-1}{3}}$

۶)
$f(x) = 9^x \,,\ g(x) = \log_3^x \longrightarrow fog(x) \leq gof(x)$

$9^{\log_3^x} \leq \log_3^{9^x} \Rightarrow x^2 \leq 2x \longrightarrow x^2 - 2x \leq 0$ $\quad \begin{array}{ccccc} & 0 & & 2 & \\ + & | & - & | & + \end{array}$

جواب: $x \in [0\,,2]$

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۷)
$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 + \frac{4}{x^2} - \frac{2}{x} \longrightarrow$

$f(\underbrace{x - \frac{2}{x}}_{t}) = (x - \frac{2}{x}) + (x - \frac{2}{x})^2 \Rightarrow f(t) = t + t^2$

$\Rightarrow f(x) = x^2 + x$

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۸)
$f(x) = \log_2^x \,,\ g(x) = 8x - x^2 \Rightarrow fog(x) = \log_2^{8x - x^2}$

$D_{fog} : 8x - x^2 > 0 \longrightarrow \begin{array}{ccccc} & 0 & & 8 & \\ - & | & + & | & - \end{array} \Rightarrow D_{fog} = (0\,,8) = A$

$R_{fog} = \mathbb{R} = B \Rightarrow A \cap B = (0\,,8) \cap \mathbb{R} \Rightarrow \boxed{A \cap B = (0\,,8)}$

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۹)
$fof(x) = 1 - 3f^2(x) \Rightarrow f(x) = 1 - 3x^2$

$h(g(x)) = f(x) \longrightarrow (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2$

$x^3 + x^2 + 4x - 3 = 0 \longleftarrow$ حداکثر ۳ ریشه دارد

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۱۰)
$f(x) = x - \frac{4}{x} \longrightarrow x - \frac{4}{x} = 3 \longrightarrow x = -1\,,4$

$gof(x) = ax^3 + 2x \xrightarrow{g(3)=8} \begin{cases} g(f(-1)) = -a - 2 = 8 \Rightarrow a = -10 \\ g(f(4)) = 64a + 8 = 8 \Rightarrow a = 0 \end{cases}$

$\boxed{a = -10 \,,\ 0}$