خورشید احمدی $\qquad$ 18/75 $\qquad$ دوازدهم دختر 13

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1) $\bullet f(x)=\sqrt{\log_2^{(x-1)}} \xrightarrow{D_f} \log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2 \to D_f:[2,+\infty)$

2

$R_f=[0,+\infty)$

$\bullet g(x)=\sqrt{-x^2+4x-4} \xrightarrow{D_g} -x^2+4x-4 \geq 0 \to -(x-2)^2 \geq 0 \to D_g=\{2\}$

$\bullet D_{gof}:\{x \in D_f \,,\, R_f \in D_g\} \to \{x \in [2,+\infty) \,,\, [0,+\infty) \in \{2\}\}$

$\xrightarrow{x-1=16} D_{gof}:\{17\}$

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2) $fog(x)=gof(x) \Rightarrow 2(x^2-3x+8)-5=(2x-5)^2-3(2x-5)+8$

$\Rightarrow 2x^2-6x+16-5=4x^2+25-20x-6x+15+8$

2

$\Rightarrow 2x^2-20x+37=0 \longrightarrow S=10 \,,\, P=\frac{37}{2}$

$\alpha^2+\beta^2=S^2-2P=100-37=\boxed{43}$

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3) $gof(x)=5x^2+11 \,,\, f(x)=2x$

2

$\Rightarrow g(2x)=5x^2+11 \longrightarrow g(x)=5(2x)^2+11$

$\Rightarrow g(x)=20x^2+11 \longrightarrow g(x-7)=20\underbrace{(x-7)^2}_{\text{0 ← برای کمترین مقدار}}+11$

$\boxed{\min_{g(x-7)}=11}$

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4) $f(x)=3x-4 \,,\, fog(x)=3x^2-6x-1$

$\Rightarrow f(g(x))=3(g(x))-4=3x^2-6x-1 \Rightarrow g(x)=x^2-2x+1$

2

$\xrightarrow{gof(x)} (3x-4)^2-2(3x-4)+1 \Rightarrow \boxed{gof(x)=9x^2-30x+25}$

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5) $(f-g)(x)=-x+5 \,,\, 14=2\underbrace{(f(x)-g(x))}_{(f-g)(x)}+f(x)$

2

$\Rightarrow f(x)=2x+4 \,,\, 14=3f(x)-2g(x) \longrightarrow g(x)=3x-1$

$fog(x)=\frac{-2}{a}x+2=2(3x-1)+4 \Rightarrow \boxed{a=\frac{-1}{3}}$

۶)

$f(x)=9^x \,,\, g(x)=\log_3^x \longrightarrow fog(x)\le gof(x)$

$9^{\log_3^x} \le \log_3^{9^x} \Rightarrow x^2 \le 2x \longrightarrow x^2-2x\le 0$ $\quad \begin{array}{ccccc} & 0 & & 2 & \\ + & | & - & | & + \end{array}$

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جواب: $x\in[0,2]$

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۷)

$f\left(\frac{x^2-2}{x}\right)=x^2+x-4+\frac{4}{x^2}-\frac{2}{x} \longrightarrow$

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$f(\underbrace{x-\frac{2}{x}}_{t})=(x-\frac{2}{x})+(x-\frac{2}{x})^2 \Rightarrow f(t)=t+t^2$

$\Rightarrow f(x)=x^2+x$

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۸)

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$f(x)=\log_2^x \,,\, g(x)=8x-x^2 \Rightarrow fog(x)=\log_2^{8x-x^2}$

$D_{fog}: 8x-x^2>0 \longrightarrow \begin{array}{ccccc} & 0 & & 8 & \\ - & | & + & | & - \end{array} \Rightarrow \underbrace{D_{fog}=(0,8)=A}$

$\underbrace{R_{fog}=\mathbb{R}=B} \Rightarrow A\cap B=(0,8)\cap\mathbb{R} \Rightarrow \boxed{A\cap B=\cancel{(0,8)}}$ $(0,4]$

$R_g=(-\infty,16] \xrightarrow{\log_2} (-\infty,4] \rightarrow R_{fog}=(-\infty,4]$

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۹)

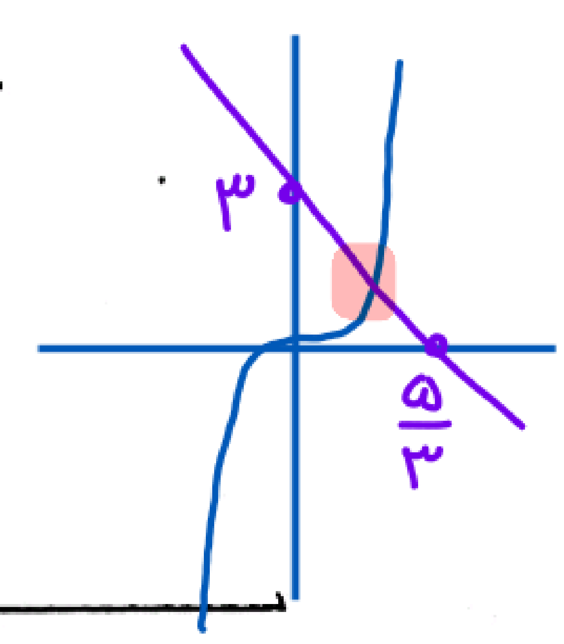
۱٫۵

$fof(x)=1-3f^2(x) \Rightarrow f(x)=1-3x^2$

$h(g(x))=f(x) \longrightarrow (x-1)^3+2(x-1)+1=1-3x^2$

$x^3+x^{\cancel{2}}+4x-3=0 \longleftarrow$ حداکثر ~~۳~~ ریشه دارد ۱

$x^3=-5x+3 \longrightarrow$



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۱۰)

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$f(x)=x-\frac{4}{x} \longrightarrow x-\frac{4}{x}=3 \longrightarrow \underbrace{x=-1,4}$

$gof(x)=ax^3+2x \xrightarrow{g(3)=8} \begin{cases} g(f(-1))=-a-2=8\Rightarrow a=-10 \\ g(f(4))=64a+8=8\Rightarrow a=0 \end{cases}$

$\boxed{a=-10 \text{ و } 0}$