

سوال ۱)

$$f(n) = \sqrt[n]{\log(n-1)} \quad g(n) = \sqrt[n]{-n^2 + 4n - 4} \quad D_{g \circ f} = \{x \in D_f : f(n) \in D_g\} = \{x \geq 2 \cap x = 17\} = \{17\}$$

$\downarrow$
 $n-1 > 0 \rightarrow n > 1$ ① $\log(n-1) > 0 \rightarrow n-1 \geq 1 \rightarrow n \geq 2$ ② $\Rightarrow D_f = ① \cap ② = n \geq 2$
 $-n^2 + 4n - 4 \geq 0 \rightarrow (n-2)^2 \leq 0 \rightarrow \frac{2}{-4 \pm 4} \Rightarrow D_g = \{2\}$
 $\rightarrow f(n) = 2 \rightarrow \sqrt[n]{\log(n-1)} = 2 \rightarrow \log(n-1) = 2^n$
 $n-1 = 17 \rightarrow n = 18$

سوال ۲)

$$f(n) = 2n - 5 \quad g(n) = n^2 - 3n + 1 \quad f \circ g(n) = 2(n^2 - 3n + 1) - 5 = 2n^2 - 4n - 3$$

$$g \circ f(n) = (2n - 5)^2 - 3(2n - 5) + 1 = 4n^2 - 20n + 25 - 6n + 15 + 1 = 4n^2 - 26n + 41$$

$$f \circ g(n) = g \circ f(n) \rightarrow 2n^2 - 4n - 3 = 4n^2 - 26n + 41 \rightarrow 2n^2 - 22n + 44 = 0 \rightarrow \frac{22}{2} \pm \sqrt{\frac{22^2}{4} - 4 \cdot 2 \cdot 44} = 11 \pm 11 \rightarrow n = 11 \text{ or } 2$$

سوال ۳)

$$g \circ f(n) = 5n^2 + 11 \quad f(n) = 2n \quad 2n = t \rightarrow n = \frac{t}{2} \rightarrow g(t) = 5\left(\frac{t}{2}\right)^2 + 11 = \frac{5t^2}{4} + 11 \rightarrow g(n) = \frac{5}{4}n^2 + 11$$

$$g(n-1) = \frac{5}{4}(n-1)^2 + 11 = \frac{5}{4}n^2 - \frac{5}{2}n + \frac{5}{4} + 11 = \frac{5}{4}n^2 - \frac{5}{2}n + \frac{47}{4}$$

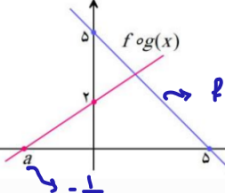
$$x = -\frac{b}{2a} = -\frac{-\frac{5}{2}}{\frac{5}{4}} = 2 \rightarrow \frac{5}{4}(2-1)^2 + 11 = \frac{5}{4} + 11 = \frac{49}{4}$$

سوال ۴)

$$f(n) = 3n - 4 \quad f \circ g(n) = 3n^2 - 4n - 1 \quad f(g) = 3(g) - 4 \rightarrow 3(g) - 4 = 3n^2 - 4n - 1 \rightarrow g(n) = \frac{3n^2 - 4n - 1}{3} = n^2 - \frac{4}{3}n - \frac{1}{3}$$

$$g \circ f(n) = (3n - 4)^2 - 2(3n - 4) + 1 = 9n^2 - 24n + 16 - 6n + 8 + 1 = 9n^2 - 30n + 25 = (3n - 5)^2$$

سوال ۵)



$$2g(n) - 3f(n) = -12 \quad \begin{cases} 2g - 3f = -12 \\ -2g + 2f = -2n + 10 \end{cases} \rightarrow -f(n) = -2n - 1 \rightarrow f(n) = 2n + 1 \Rightarrow g(n) = 3n - 1$$

$$f \circ g(n) = \frac{1}{2}n + 2 \Rightarrow \frac{1}{2}n = 4 \rightarrow n = 8 \rightarrow a \text{ is } 8$$

سوال ۶)

$$f(n) = 9^n \quad g(n) = \log_9 n \quad f \circ g(n) = 9^{\log_9 n} = n \quad g \circ f(n) = \log_9 9^n = n$$

$$f \circ g(n) \neq g \circ f(n) \rightarrow n^2 \neq 2n \rightarrow n^2 - 2n \neq 0 \rightarrow \frac{0}{-1 \pm 1} \rightarrow 0 < n < 2 \rightarrow n = \{1, 2\} \rightarrow n \in \mathbb{N}$$

سوال ۷)

$$f\left(\frac{n^2 - 2}{n}\right) = n^2 + n - 4 - \frac{2}{n} + \frac{4}{n^2} \rightarrow f\left(n - \frac{2}{n}\right) = n^2 + n - 4 - \frac{2}{n} + \frac{4}{n^2} \rightarrow f\left(n - \frac{2}{n}\right) = n^2 + n - 4 - \frac{2}{n} + \frac{4}{n^2}$$

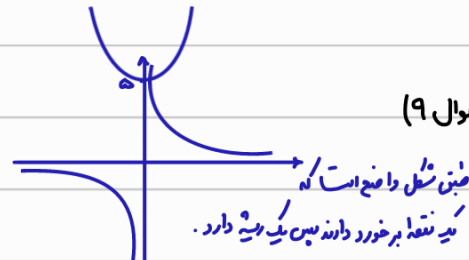
سوال ۸)

$$f(n) = \log_2 n \quad g(n) = 12n - n^2 \quad f \circ g(n) = \log_2(12n - n^2) \quad D_{f \circ g} = \{n \in D_g : g(n) \in D_f\} = \{n \in \mathbb{R} \cap (0, 12)\} = (0, 12) = A$$

$$g(n) > 0 \rightarrow 12n - n^2 > 0 \rightarrow n^2 - 12n < 0 \rightarrow \frac{0}{-1 \pm 1} \rightarrow 0 < n < 12 \rightarrow \log_2(12n - n^2) \xrightarrow{\text{max}} n = \frac{12}{2} = 6 \rightarrow \log_2(12 \cdot 6 - 6^2) = \log_2 36 = 5 \Rightarrow R_{f \circ g} = (-\infty, 5] = B$$

$$A \cap B = (0, 12) \cap (-\infty, 5] = (0, 5] \rightarrow \{1, 2, 3, 4, 5\} \rightarrow \text{مجموعه صحیح}$$

سوال ۹



$$f(f(x)) = 1 - 3f'(x) \rightarrow f(x) = 1 - 3x^2$$

$$h(g(x)) = (x-1)^2 + 2(x-1) + 1 = x^2 - 2x^2 + 2x - 1 + 2x - 2 + 1 = x^2 - 2x^2 + 4x - 2$$

$$h(g(x)) = f(x) \rightarrow 1 - 3x^2 = x^2 - 2x^2 + 4x - 2 \rightarrow x^2 + 4x = 3 \rightarrow x(x+4) = 3 \rightarrow x^2 + 4x = \frac{3}{x}$$

$$f(x) = x - \frac{x}{x} \quad g \circ f(x) = ax^2 + 2x \quad g(3) = 1 \quad x - \frac{x}{x} = 3 \xrightarrow{x \neq 0} x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \xrightarrow{x=4} x=4 \quad \text{سوال ۱۰}$$

$$x=4 \rightarrow a(4)^2 + 2(4) = 1 \rightarrow 42a = 0 \rightarrow a = 0$$

$$x=-1 \rightarrow a(-1)^2 + 2(-1) = 1 \rightarrow -a = 1 \rightarrow a = -1$$