

سوال ۱) $f(n) = \sqrt[n]{\log(n-1)}$ $g(n) = \sqrt[n^2+4n-4}$ $D_{g \circ f} = \{x \in D_f : f(n) \in D_g\} = \{x \geq 2 \cap x = 17\} = \{17\}$

$\downarrow$
 $n-1 > 0 \rightarrow n > 1$ ① $\log(n-1) > 0 \rightarrow n-1 \geq 1 \rightarrow n \geq 2$ ② $\Rightarrow D_f = ① \cap ② = n \geq 2$

$-n^2+4n-4 > 0 \rightarrow (n-2)^2 < 0$ $\frac{1}{4} \Rightarrow D_g = \{2\}$

$\rightarrow f(n) = 2 \rightarrow \sqrt[n]{\log(n-1)} = 2 \rightarrow \log(n-1) = 2^n$
 $n-1 = 17 \rightarrow n = 18$

سوال ۲) $f(n) = 2n-5$ $g(n) = n^2-3n+8$ $f \circ g(n) = 2(n^2-3n+8)-5 = 2n^2-4n+11$

$g \circ f(n) = (2n-5)^2 - 3(2n-5) + 8 = 4n^2 - 20n + 25 - 6n + 15 + 8 = 4n^2 - 26n + 48$

$f \circ g(n) = g \circ f(n) \rightarrow 2n^2 - 4n + 11 = 4n^2 - 26n + 48 \rightarrow 2n^2 - 22n + 37 = 0$ $\frac{22}{2} \pm \sqrt{\frac{22^2}{4} - 4 \cdot 2 \cdot 37} = 11 \pm \sqrt{121 - 304} = 11 \pm \sqrt{-183}$

سوال ۳) $g \circ f(n) = 5n^2+11$ $f(n) = 2n$ $2n = t \rightarrow n = \frac{t}{2} \rightarrow g(t) = 5(\frac{t}{2})^2 + 11 = \frac{5t^2}{4} + 11 \rightarrow g(n) = \frac{5}{4}n^2 + 11$

$g(n-v) = \frac{5}{4}(n-v)^2 + 11 = \frac{5}{4}n^2 - \frac{5v}{2}n + \frac{5v^2}{4} + 11$ $x = -\frac{b}{a} = -\frac{-\frac{5v}{2}}{\frac{5}{4}} = v$ $x=v \rightarrow \frac{5}{4}(v^2) - \frac{5v}{2}v + \frac{5v^2}{4} + 11 = \frac{5v^2}{4} - \frac{5v^2}{2} + \frac{5v^2}{4} + 11 = 11$

سوال ۴) $f(n) = 3n-4$ $f \circ g(n) = 3n^2-4n-1$ $f(g) = 3(g)-4 \rightarrow 3(g)-4 = 3n^2-4n-1 \rightarrow g(n) = \frac{3n^2-4n+3}{3} = n^2-2n+1$

$g \circ f(n) = (3n-4)^2 - 2(3n-4) + 1 = 9n^2 - 24n + 16 - 6n + 8 + 1 = 9n^2 - 30n + 25 = (3n-5)^2$

سوال ۵) $2g(n) - 3f(n) = -15$ $2g - 3f = -15$ $f \circ g(n) = 2(3n-1) + 5 = 6n+2$

$-f(n) = -3n-5 \rightarrow f(n) = 3n+5 \Rightarrow g(n) = 3n-1$

$f \circ g(n) = \frac{3}{2}n+2 \Rightarrow \frac{3}{2}n = 4 \rightarrow n = \frac{8}{3} \rightarrow n$ صحیح

سوال ۶) $f(n) = 9^n$ $g(n) = \log_9 n$ $f \circ g(n) = 9^{\log_9 n} = n$ $g \circ f(n) = \log_9 9^n = n$

$f \circ g(n) \neq g \circ f(n) \rightarrow n^2 \neq 2n \rightarrow n^2 - 2n < 0 \rightarrow \frac{0}{1} - \frac{2}{1} + \frac{1}{1} \rightarrow 0 < n < 2 \rightarrow n = \{1, 2\} \rightarrow n \in \mathbb{N}$

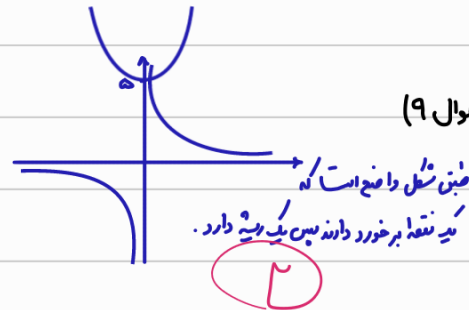
سوال ۷) $f\left(\frac{n^2-2}{n}\right) = n^2+n-4-\frac{2}{n}+\frac{4}{n^2} \rightarrow f\left(n-\frac{2}{n}\right) = n-\frac{2}{n}+n^2-4+\frac{4}{n^2} \rightarrow f\left(n-\frac{2}{n}\right) = n-\frac{2}{n}+\left(n-\frac{2}{n}\right)^2 \rightarrow f(n) = n^2+n$

سوال ۸) $f(n) = \log_9 n$ $g(n) = 11n-n^2$ $f \circ g(n) = \log_9(11n-n^2)$ $D_{f \circ g} = \{n \in D_g : g(n) \in D_f\} = \{n \in \mathbb{R} \cap (0, 11)\} = (0, 11) = A$

$g(n) > 0 \rightarrow 11n-n^2 > 0 \rightarrow n^2-11n < 0$ $\frac{0}{1} - \frac{11}{1} + \frac{1}{1} \rightarrow 0 < n < 11$ $\log_9(11n-n^2) \xrightarrow{\max} x = \frac{11}{2} = 5.5 \rightarrow \log_9(11n-n^2) \xrightarrow{\max} \log_9 30.25 = 4 \Rightarrow R_{f \circ g} = (-\infty, 4] = B$

$A \cap B = (0, 11) \cap (-\infty, 4] = (0, 4] \rightarrow \{1, 2, 3, 4\} \rightarrow$ صحیح

سوال ۹



$$f(f(x)) = 1 - 3f'(x) \rightarrow f(x) = 1 - 3x^2$$

$$h(g(x)) = (x-1)^2 + 2(x-1) + 1 = x^2 - 2x^2 + 4x - 1 + 2x - 2 + 1 = x^2 - 3x^2 + 6x - 2$$

$$h(g(x)) = f(x) \rightarrow 1 - 3x^2 = x^2 - 3x^2 + 6x - 2 \rightarrow x^2 + 6x = 3 \rightarrow x(x+6) = 3 \rightarrow x+6 = \frac{3}{x}$$



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$$f(x) = x - \frac{x}{x} \quad g \circ f(x) = ax^2 + 2x \quad g(3) = 1$$

$$x - \frac{x}{x} = 3 \xrightarrow{x \neq 0} x^2 - 3x - 6 = 0 \rightarrow (x-6)(x+1) = 0 \xrightarrow{x=4} x=-1$$

سوال ۱۰

$$x=6 \rightarrow a(6)^2 + 2(6) = 1 \rightarrow 42a = 0 \rightarrow a=0$$



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$$x=-1 \rightarrow a(-1)^2 + 2(-1) = 1 \rightarrow -a = 1 \rightarrow a = -1$$