مهسا بخاتی / تکلیف شماره‌ی ۷

(۱)

$y = -\frac{1}{3}x^2 + \frac{2}{3}x + 1$

$x = \frac{-b}{2a} = \frac{-\frac{2}{3}}{-\frac{2}{3}} = 1$

$-\frac{1}{3} \times 1^2 + \frac{2}{3} \times 1 + 1 = \frac{28}{27} = 1\frac{4}{27}$

$\frac{3}{3} + \frac{17}{3} = 4 \qquad 1\frac{3}{27} + 1\frac{22}{27} = 3$

$y' = -\frac{1}{3}\left(x + \frac{17}{3}\right)^2 + \frac{2}{3}\left(x + \frac{17}{3}\right) + 1 + 1\frac{22}{27}$

$x^2 + \frac{34x}{3} + \frac{289}{27}$

$y' = -\frac{x^2}{3} + \frac{34x}{13} - \frac{289}{73} + \frac{2x}{3} + \frac{34}{27} + 1 + \frac{47}{27}$

$y' = -\frac{x^2}{3} + \left(\frac{2}{13} - \frac{34}{13}\right)x - \frac{289}{73} + \frac{102}{73} + \frac{73}{73} + \frac{141}{73}$

$y' = -\frac{x^2}{3} + \left(\frac{-28}{13}\right)x + \frac{29}{73} \rightarrow y' = 0$

$-\frac{x^2}{3} + \frac{-28}{13}x + \frac{29}{73} = 0 \xrightarrow{\times 73} -27x^2 - 140x + 29 = 0$

$27x^2 + 140x + 29 = 0$

صادقی نهمن برحسب رأس و ضریب آن $y = -\frac{1}{3}(x-4)^2 + 3$

$y = 0 \rightarrow -\frac{1}{3}(x-4)^2 + 3 \rightarrow (x-4)^2 = 9 \rightarrow x < 7$ ✓

یا $x < 1$ ✓

(۲) $y = 1 - 2f(3x+2)$

$3x+2 = 1 \to 3x = -1 \to x = -\frac{1}{3}$

$-2 \times 4 = -8 \to -8+1 = -7$

$\Rightarrow A(\alpha, \beta) = (-\frac{1}{3}, -7)$

$\alpha\beta = -\frac{1}{3}, -7 = \frac{7}{3}$ ✓

(۳) $y = 3 - 4\sqrt{2x}$

x های داخل رادیکال $\to 3 - \sqrt{2(x+1)}$

نسبت به x ها قرینه $\to -(3 - \sqrt{2(x+1)}) = \sqrt{2(x+1)} - 3$

a واحد در جهت مثبت y ها $\to \sqrt{2(x+1)} + 2$ $\xrightarrow{y=\frac{7}{2}}$

$\frac{7}{2} - 2 = \sqrt{2(x+1)} \Rightarrow \frac{9}{4} = 2(x+1) \to \frac{9}{8} = x+1 \to x = \frac{1}{8}$ ✓

(۴) $D_f = (a, 4]$ $D_g = [-1, b)$

$y = f(x-1) + g(x+1) \to D_y = (-2, 3)$

$D_{f(x-1)} = (a+1, 3]$ ، $D_{g(x+1)} = [-2, b-1)$

$[-2, b-1) \cap (a+1, 3] = (-2, 3)$

$b-1 = 3 \to b = 4$

$a+1 = -2 \to a = -3$

$a - b = -3 - 4 = -7$ ✓

(۵) $y_1 = f(\frac{x}{2}+1) \to D_f = [-4, 3]$ ، $R_f = [-1, 2]$

$x = -4 \to -\frac{4}{2} + \frac{2}{2} = -\frac{1}{2}$ $\quad x = 3 \to \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$

$\Rightarrow D_{f(x)} = [-\frac{1}{2}, \frac{5}{2}]$ ، $R_{f(x)} = [-1, 2]$ ✓

$y_2 = \sqrt{|2f(x) - 3|}$

$D_{y_2} = [-\frac{1}{2}, \frac{7}{2}]$, $R_{y_2} = [\sqrt{|(-1\times 2) - 3|}, \sqrt{|(2\times 2) - 3|}] = [1, \sqrt{5}]$

سوال ۴ - ۵

نقاط توخالی و توپر برعکس هستند ← $a<0$

$f(-\frac{1}{a}) = -\frac{1}{a^2}[-1] = \frac{1}{4} \rightarrow a^2 = 4 \xrightarrow{a<0} a = -2$

$b = 1 \rightarrow \boxed{b - a = 3}$

(۷) (۱٫۵)

$f(x) = \log_2 x$

۲ واحد به سمت چپ → $\log_2 (x+2)$

نسبت به محور xها قرینه → $-\log_2 (x+2)$

۳ واحد بالا → $-\log_2 (x+2) + 3$

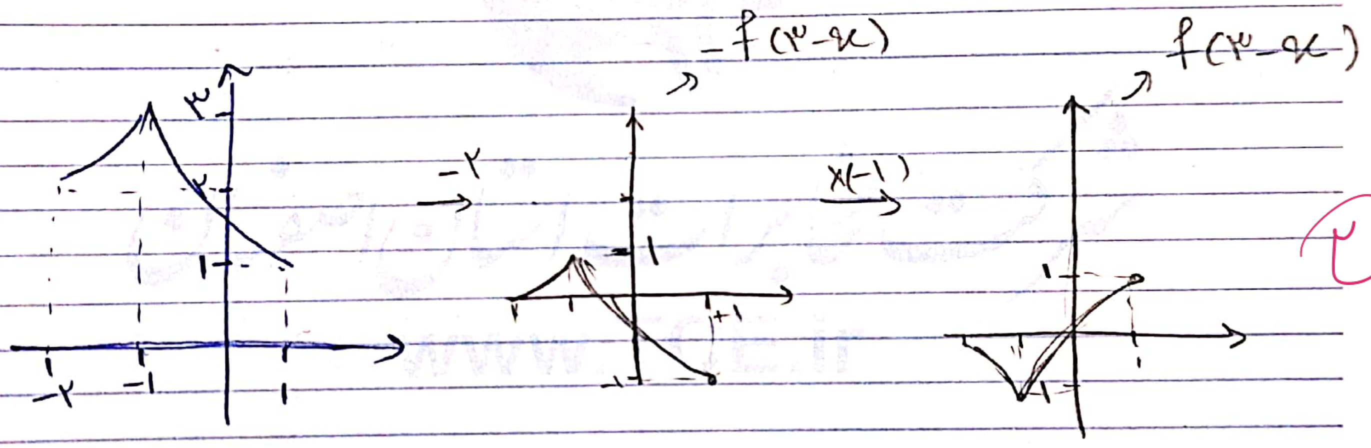
نسبت به محور yها → $-\log_2 (-x+2) + 3 = g(x)$

$g(-14) + g(-62) = -\log_2 16 - \log_2 64 = -4 - 6 = \sout{-10}$

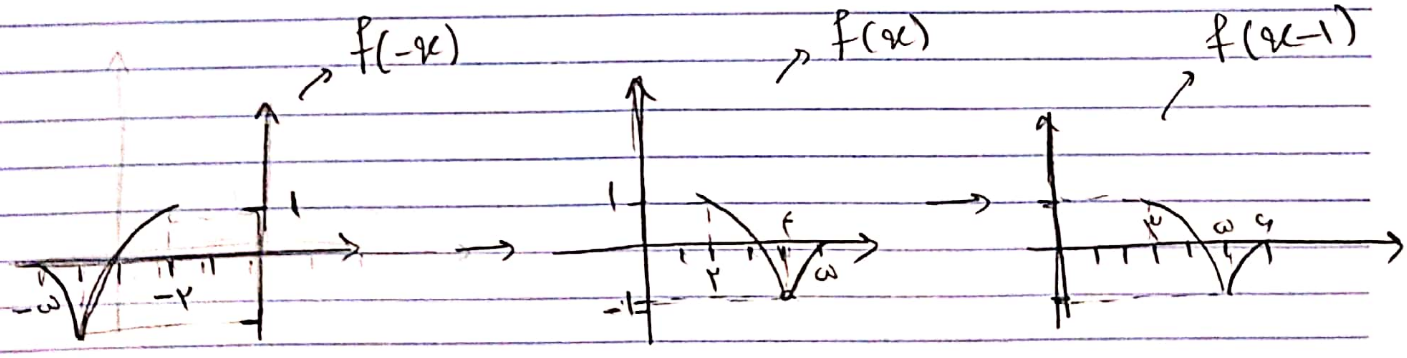
چرا (۳) را یادت رفت !! $-4 + 3 - 6 + 3 = -4$

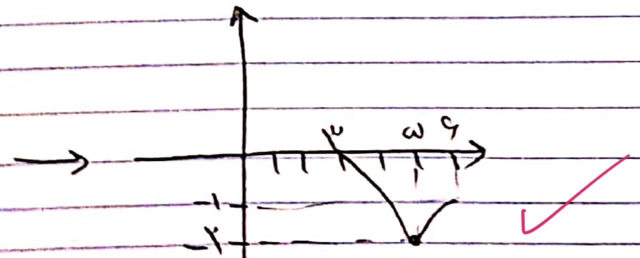
(۸)

$y_1 = 2 - f(3-x) \qquad y_2 = f(x-1) - 1$



(۲)





(۹)

$$f(x)=\left|\frac{x+1}{2x+1}\right|$$

۱/۲۵

۲ واحد به سمت راست $\rightarrow \left|\frac{x-2+1}{2(x-2)+1}\right| = \left|\frac{x-1}{2x-3}\right|$

یک واحد به سمت بالا $\rightarrow \left|\frac{x-1}{2x-3}\right|+1 = g(x)$

$$x+\left|\frac{x-1}{2x-3}\right|+1<0 \rightarrow \left|\frac{x-1}{2x-3}\right| < -1-x$$

جواب قدر مطلق باید مثبت باشد $\Rightarrow x<-1$

x	$-\infty$		-1		$\frac{3}{2}$		$+\infty$
$x+1+\frac{-x+1}{2x-3}$		+	۰	−	‖		
$x+1+\frac{x-1}{2x-3}$					‖	+	

$$x+1+\frac{x-1}{2x-3}<0 \rightarrow \frac{2x^2-x-3+x-1}{2x-3}<0$$

$$\frac{2x^2-x-3-x+1}{2x-3} \text{ یا } \frac{2x^2-4}{2x-3}<0$$

دقت کن!

x	$-\infty$		$-\sqrt{2}$		$\frac{3}{2}$		$\sqrt{2}$		$+\infty$
$2x^2-4$		+	۰	−		−	۰	+	
$2x-3$		−		−	۰	+		+	

$$\frac{2x^2-2x-2}{2x-3}<0$$

$x<-\sqrt{2}$ ← جواب

$$g(x) = \sqrt{28f(x) - f^2(x)}$$

(۱۰) ۰۸۵

$$y = x^3 \rightarrow y = (x-2)^3 + 27 = f(x)$$

$$-f^2(x) + 28f(x) \geq 0 \xrightarrow{f(x)=t} -t^2 + 28t \geq 0 \rightarrow -t(t-28) \geq 0$$

$$\begin{array}{c|ccc} & 0 & & 28 \\ \hline & - & + & - \end{array} \rightarrow 0 \leq f(x) \leq 28 \rightarrow 0 \leq (x-2)^3 + 27 \leq 28 \xrightarrow{-27}$$

$$-27 \leq (x-2)^3 \leq 1 \xrightarrow{\sqrt[3]{\ }} -9 \leq x-2 \leq 1 \xrightarrow{+2} -7 \leq x \leq 3$$

$$D_f = [-7, 3]$$

$$f(n) = k(n-2)^3 + 27 \xrightarrow{(1,0)} k < \frac{27}{8}$$

$$0 \leq f(n) \leq n \rightarrow f(n) \in \mathbb{N}_0 \rightarrow n \in \mathbb{N}_0$$

$$\hookrightarrow f(n) < n \rightarrow n \in \frac{\Delta}{\in} \rightarrow 0 \leq n \leq \frac{\Delta}{\in}$$

۳ عدد صحیح