

1 $y = \frac{1}{3} n^2 + \frac{2}{3} n + 1 \rightarrow$ فرض کنیم: $\frac{b}{2a} = \frac{-\frac{2}{3}}{\frac{2}{3}} = -1 \Rightarrow$ فرض کنیم: $y = \frac{1}{3} (n-1)^2 + \frac{2}{3} n \Rightarrow y = \frac{1}{3} (n-1)^2 + \frac{2}{3} n$
 $g(n) = a(n-1)^2 + 3$ از اینجا می بینیم که تابع به صورت مربع کامل در می آید $a = \frac{1}{3} \Rightarrow g(n) = \frac{1}{3} (n-1)^2 + 3 \rightarrow \frac{1}{3} (n-1)^2 + 3 = 0 \Rightarrow (n-1)^2 = -9 \Rightarrow n-1 = \pm 3$ n=1 n=7

2 $B: (1, 4) \rightarrow A: (\alpha, \beta) \quad f(n+2) = f(1) \Rightarrow \alpha = \frac{1}{3} = \alpha \quad 1-2f(1) = 1-2(4) = -7 = \beta \quad \alpha/\beta = \frac{1}{3} \div -7 = -\frac{1}{21}$

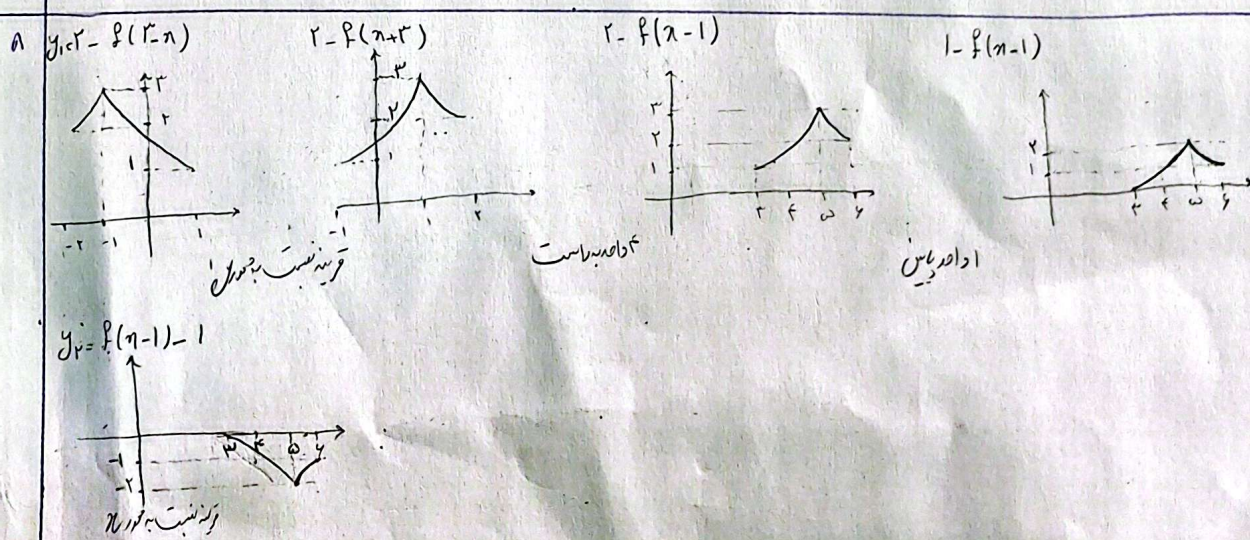
3 $y = 2 - \sqrt{2n} \rightarrow f(n) = 2 - \sqrt{2(n+1)} = 2 - \sqrt{2n+2} \rightarrow g(n) = 2 - \sqrt{2n+2} \rightarrow h(n) = \sqrt{2n+2} + 2$
در ادامه جهت مثبت ی
 $\sqrt{2n+2} + 2 = \frac{1}{3} \Rightarrow \sqrt{2n+2} = \frac{1}{3} - 2 = -\frac{5}{3} \Rightarrow 2n+2 = \frac{25}{9} \Rightarrow 2n = \frac{25}{9} - 2 = \frac{25-18}{9} = \frac{7}{9} \Rightarrow n = \frac{7}{18}$

4 $D_f = (a, 2) \rightarrow a+1 < 2 \Rightarrow a < 1$ $D_g = [-1, b) \rightarrow -1 < n+1 < b \Rightarrow -2 < n < b-1$
 $D_f \cap D_g = (a+1, 2) \cap [-2, b-1) = (-2, 2) \Rightarrow a+1 = -2 \Rightarrow a = -3$
 $b-1 = 2 \Rightarrow b = 3 \rightarrow a-b = -3-3 = -6$

5 $2 < x < 5 \Rightarrow \frac{2}{3} < \frac{x}{3} < \frac{5}{3} \Rightarrow \frac{2}{3} < \frac{x}{3} + 1 < \frac{5}{3} + 1 \rightarrow D_f: [\frac{5}{3}, \frac{8}{3}]$
 $-1 < y < 2 \Rightarrow R_f: [-1, 2]$
 $y = \sqrt{12f(n)-5}$
 $12f(n)-5 \geq 0 \Rightarrow 12f(n) \geq 5 \Rightarrow f(n) \geq \frac{5}{12}$
 $-1 < f(n) < 2 \Rightarrow -2 < 12f(n) < 24 \Rightarrow 0 < 12f(n)-5 < 19 \Rightarrow 0 < \sqrt{12f(n)-5} < \sqrt{19} \Rightarrow R_{y_2} = [0, \sqrt{19}]$

6 $y = \frac{n}{a} [an]$ $a \in \mathbb{N} \rightarrow [an] \in \mathbb{N} \Rightarrow y \in \mathbb{N}$
 $y = \frac{1}{a} [an] \Rightarrow \frac{1}{a} \leq y \leq \frac{a}{a} = 1 \Rightarrow y \in [\frac{1}{a}, 1]$
 $\Rightarrow b = a-1 = (-2) = -3$

7 $f(n) = \log_2 n \rightarrow h(n) = \log_2 (n+2) \rightarrow p(n) = \log_2 (n+2) \rightarrow T(n) = -\log_2 (n+2) + 3 \rightarrow g(n) = -\log_2 (n+2) + 3$
 $g(1) = -\log_2 (1+2) + 3 = -1$ $g(4) = -\log_2 (4+2) + 3 = -3$ $g(1) + g(4) = (-1) + (-3) = -4$



$$f(n) = \left| \frac{n+1}{r(n+1)} \right| \rightarrow h(n) = \left| \frac{(n-2)+1}{r(n-2)+1} \right| = \left| \frac{n-1}{r(n-2)+1} \right| \rightarrow g(n) = \left| \frac{n-1}{r(n-2)+1} \right| + 1$$

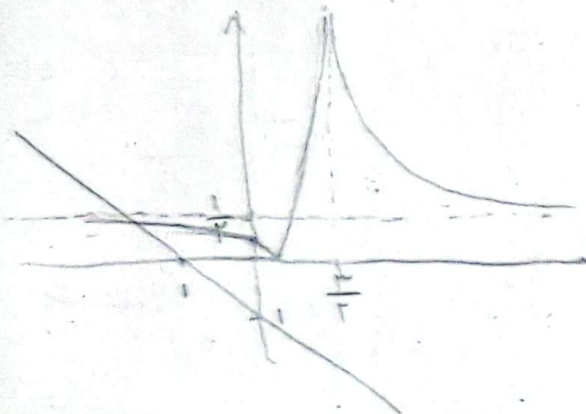
$$n + g(n) < 0 \Rightarrow n + \left| \frac{n-1}{r(n-2)+1} \right| + 1 < 0 \Rightarrow \left| \frac{n-1}{r(n-2)+1} \right| < -1 - n$$

+ $\frac{n-1}{r(n-2)+1}$
 $\frac{n-1}{r(n-2)+1} < -1 - n$

$$\frac{n-1}{r(n-2)+1} < -1 - n \Rightarrow n-1 > -r(n-2)+1 \Rightarrow r(n-2)+1 > 0$$

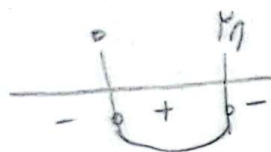
$$\Rightarrow n^2 > 2 \rightarrow n > \sqrt{2}$$

$$\Rightarrow (-\infty, -\sqrt{2})$$



$$g(n) = \sqrt{r(n)} f(n) - f'(n)$$

$$r(n) f(n) - f'(n) > 0 \xrightarrow{f(n) = t} -t^2 + r(n)t > 0$$



$$0 < t < r(n) \rightarrow 0 < f(n) < r(n)$$