[illegible]

ـ۱

$$y=-\frac{1}{3}x^2+\frac{2}{5}x+1 \rightarrow \frac{-b}{2a}=\frac{-\frac{2}{5}}{-\frac{2}{3}}=\frac{3}{5}$$

$$\rightarrow -\frac{1}{3}\left(\frac{3}{5}\right)^2+\frac{2}{5}\times\frac{3}{5}+1=\frac{-3}{25}+\frac{6}{25}+1=\frac{28}{25}$$

$$\left|\begin{matrix}\frac{3}{5}\\ \frac{28}{25}\end{matrix}\right. \rightarrow \left|\begin{matrix}4\\ 3\end{matrix}\right.$$

$$4-\frac{3}{5}=\frac{17}{5}$$

$$\Delta=b^2-4ac=\frac{4}{25}-4(1)\left(\frac{-1}{3}\right)=\frac{112}{75}$$

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{\frac{-2}{5}\pm\sqrt{\frac{112}{75}}}{\frac{-2}{3}}+\frac{17}{5}$$

طول نقطه‌ی برخورد

ـ۲

$$A(\alpha,\beta) \rightarrow y=1-2f(3x+2)$$

$$f(x) \longrightarrow \left|\begin{matrix}1\\ 4\end{matrix}\right.$$

رأس $f(x)$

$$3x+2=1 \rightarrow x=\frac{-1}{3}=\alpha$$

$$1-2\underbrace{f(1)}_{4}=\beta$$

$$1-8=\beta \rightarrow \boxed{\beta=-7}$$

$$\alpha\beta=\frac{-1}{3}\times 7=\frac{7}{3}$$

جواب

ـ۳

$$y=3-\sqrt{2x} \longrightarrow 3-\sqrt{2(x+1)}=3-\sqrt{2x+2} \longrightarrow$$

قرینه نسبت به محور x ها

$$\sqrt{2x+2}-3 \longrightarrow \sqrt{2x+2}-3+5 \longrightarrow \sqrt{2x+2}+2$$

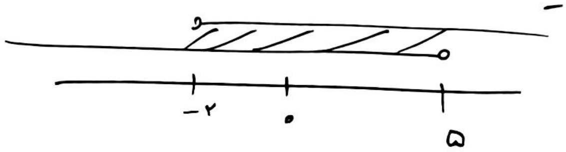
جواب

$$\sqrt{2x+2}+2=\frac{7}{2} \longrightarrow \sqrt{2x+2}=\frac{3}{2} \rightarrow 2x+2=\frac{9}{4} \rightarrow 2x=\frac{1}{4} \rightarrow \boxed{x=\frac{1}{8}}$$

ـ۴

$$D_f=(a,4] \longrightarrow D_{f(x-1)} \rightarrow (a+1,5]$$

$$D_g=[-1,b) \longrightarrow D_{g(x+1)} \rightarrow [-2,b-1)$$



$$a+1=-2 \longrightarrow \boxed{a=-3}$$

$$b-1=5 \rightarrow \boxed{b=6}$$

$$a-b=-3-6=-9$$

جواب

نکات: $f\left(\frac{x}{2}+1\right) \longrightarrow D : [-3,5] \quad R : [-1,2]$

۵ -

$f(x) \longrightarrow D_f \rightarrow -3 \leq x \leq 5 \rightarrow -2 \leq x+1 \leq 6 \longrightarrow \underbrace{-1 \leq \frac{x+1}{2} \leq 3}_{\text{دامنه } f(x)}$

$D_f = [-1,3]$

$R_f = [-1,2] \longrightarrow$ برد تابع تغییری نمی‌کند و همان برد تابع $f\left(\frac{x}{2}+1\right)$ است.

$y_2 = \sqrt{|2f(x)-3|} \longrightarrow D_{y_2} : |2f(x)-3| \geq 0$

طبق روابط $2f(x)-3$ تنها می‌تواند مقدار صفر داشته باشد.

$\longrightarrow 2f(x)-3 \geq 0$

$2f(x)-3 \leq 0$

$\longrightarrow 2f(x)-3=0$

$2f(x)=3 \longrightarrow f(x)=\frac{3}{2} \longrightarrow \boxed{D_{y_2} : \left\{\frac{3}{2}\right\}}$ جواب

$R_{y_2} \rightarrow -1 \leq f(x) \leq 2 \longrightarrow -2 \leq 2f(x) \leq 4 \longrightarrow -5 \leq 2f(x)-3 \leq 1$

$\longrightarrow \sqrt{[-5,1]} = \boxed{[0,1] : R_{y_2}}$ جواب

[illegible]

۷ـ
$f(x)=\log_2 x \longrightarrow \log_2 (x+2) \longrightarrow -\log_2 (x+2) \longrightarrow -\log_2 (x+2) + 3$

قرینه نسبت به محور y ها

$\boxed{-\log_2 (-x+2) + 3}$

$g(x) = 3 - \log_2 (-x+2)$

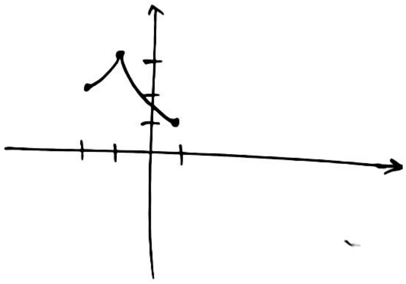
$g(-14) = 3 - \log_2 16 = 3 - 4 = -1$

$g(-62) = 3 - \log_2 64 = 3 - 6 = -3$

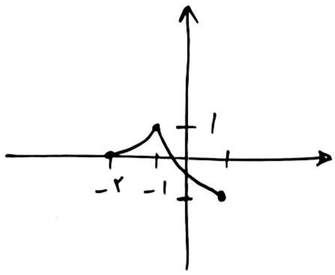
$g(-14) + g(-62) = -4$

جواب

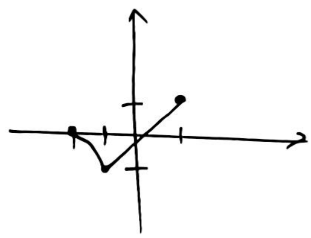
۸ـ
$y_1 = 2 - f(3-x) \longrightarrow -f(3-x) \longrightarrow f(3-x)$



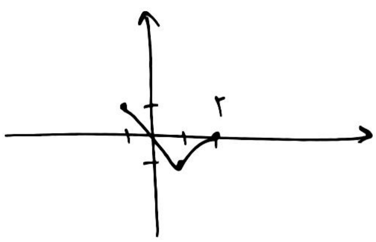
$\longrightarrow$



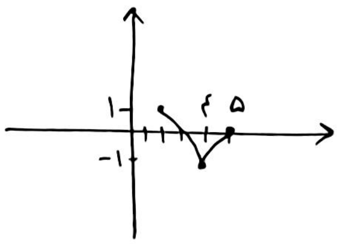
$\longrightarrow$



$\longrightarrow f(3+x) \longrightarrow f(x)$

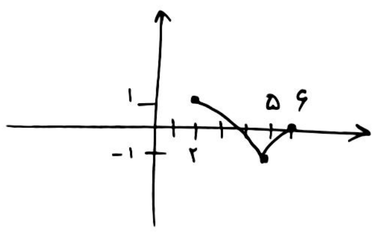


$\longrightarrow$

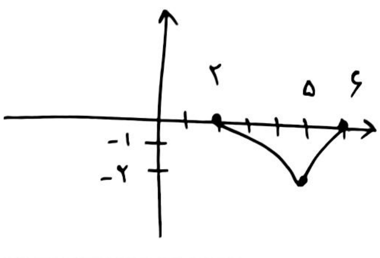


حالا از روی نمودار $f(x)$ نمودار $f(x-1)-1$ را می‌کشیم.

$\longrightarrow f(x-1) \longrightarrow f(x-1)-1$



$\longrightarrow$



$\longrightarrow$ شکل نهایی

۱۰ـ
$f(x) = (x-2)^3 + 27$

$28f(x) - f^2(x) \geq 0$

$g(x) = \sqrt{28f(x) - f^2(x)}$

$f(x)\,(28 - f(x)) = 0$

ریشه $= f(x)$
-1

$28 - (x-2)^3 - 27 = 0 \longrightarrow (x-2)^3 = 1 \longrightarrow x-2=1 \longrightarrow \boxed{x = 3}$

-1		3	
$-$	$+$		$-$

$\longrightarrow [-1, 3]$

ریشه دوم که شامل اعداد $\{-1, 0, 1, 2, 3\}$ (۵ عدد صحیح) است.