

$$f(rx+1) + f(x+r) = rx + f(r)$$

$$y = \sqrt{f(x) - x^r}$$

سوال (۱)

$$ax+b \Rightarrow a(rx+1) + b + a(x+r) + b = rx + ra + b$$

$$rax + a + ax + ra + b = rx + ra + b \Rightarrow rax + ax - rx + b = 0 \Rightarrow$$

$$rax - rx + b = 0 \Rightarrow (ra - r)x + b = 0 \Rightarrow rax + rb = rx + b$$

$$\Rightarrow rax = rx \Rightarrow \boxed{a=1} \text{ و } \boxed{b=0} \quad y=x$$

$$y = \sqrt{f(x) - x^r} \Rightarrow y = \sqrt{x - x^r} \Rightarrow \begin{cases} x_{\max} = \frac{1}{r} \\ y_{\max} = -\frac{1}{r} \end{cases} \quad \sqrt{\left[-\frac{1}{r}, \frac{1}{r}\right]}$$

$$\boxed{R_f = \left[0, \frac{1}{r}\right]}$$

سوال (۲) $f(x) = \frac{x^r - rx}{x+r}$ $R = \{a, b\}$ $f\left(\frac{a+b}{rc}\right) = ?$ $[c, +\infty) - \{1\}$

$$f(x) = \frac{x^r - rx}{x+r} \Rightarrow D_f = R - \{-r, r\} \quad \begin{cases} a = -r \\ b = r \end{cases} \quad \begin{cases} x+r \neq 0 \Rightarrow x \neq -r \\ \frac{x^r - rx}{x+r} \neq 1 \end{cases}$$

$$R_f = [c, +\infty) - \{1\}$$

$$y = \frac{x(n^r - 1)}{n+r} = \frac{x(n-r)(n+r)}{n+r} = x^r - rx \quad \begin{cases} x \neq -\frac{b}{ra} = +1 \\ y_1 = 1 - r = -1 \Rightarrow \boxed{c = -1} \end{cases}$$

$$f\left(\frac{a+b}{rc}\right) = f(-1) = (-1)^r - r(-1) = r$$

$$y = \sqrt[r]{x^r + ax^r + bx - 1} \quad D_f = [a, b] \quad R_f = ?$$

سوال (۳)

$$(x-r)^r = x^r - 4x^r + 12x - 1 \Rightarrow \begin{cases} a = -4 \\ b = 12 \end{cases}$$

$$D_f = [-4, 12]$$

$$R_f = [-1, 1]$$

$$y = \sqrt[r]{x^r + ax^r + bx - 1} = \sqrt[r]{(x-r)^r} = x-r$$

$$y_1 = \left(\frac{x+1}{1-x} \right)^{\frac{1}{5}}$$

$$y_2 = \frac{ax^2-1}{x^2+b}$$

$$\left(\frac{x+1}{1-x} \right) \geq 0 \Rightarrow \frac{-1}{-0+1} = -1 \quad \text{سؤال ٤}$$

$$Df = [-1, 1]$$

$$R_{y_2} = \begin{cases} x=0 \rightarrow -\frac{1}{b} \\ x \rightarrow \infty \rightarrow a \end{cases} \Rightarrow \begin{cases} \text{مخرج منفرج} \rightarrow (-\frac{1}{b}, a) \\ -\frac{1}{b} = -1 \Rightarrow b=1 \end{cases}$$

$$\boxed{a=1} \quad \boxed{b=1}$$

$$a+b=1+1=2$$

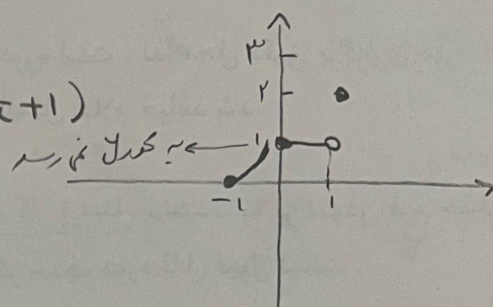
$$\frac{[x]}{x} + \frac{|x|}{x} \text{ في } [-1, 1] = ?$$

$$-1 < x < 0 \Rightarrow -\frac{1}{x} + \frac{(-x)}{x} = -\frac{1}{x} - 1 \Rightarrow -\left(\frac{1}{x} + 1\right)$$

$$0 \leq x < 1 \Rightarrow \frac{x}{x} + \frac{x}{x} = 1$$

$$x=1 \Rightarrow \frac{1}{1} + \frac{1}{1} = 1 + 1 = 2$$

$$Df = \mathbb{R} - \{0\} \quad \text{حيث } x \neq 0$$

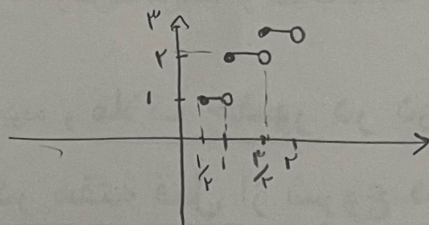


$$y = [x] + \left[x + \frac{1}{x} \right] \quad \left(\frac{1}{x}, 2 \right)$$

$$\frac{1}{x} \leq x < 1 \Rightarrow 0 + 1 = 1$$

$$1 \leq x < \frac{2}{x} \Rightarrow 1 + 1 = 2$$

$$\frac{2}{x} \leq x < 2 \Rightarrow 1 + 2 = 3$$



$$\left| \frac{x+1}{ax+3} \right| \Rightarrow \left| \frac{x+1}{ax+3} \right| < 1 \Rightarrow (x+1)^2 < (ax+3)^2$$

$$(x+1)^2 - (ax+3)^2 < 0 \Rightarrow (x^2 + 2x + 1) - (a^2x^2 + 6ax + 9) < 0 \Rightarrow$$

$$(1-a^2)x^2 + (2-6a)x - 8 < 0 \Rightarrow 1-a^2 = 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1 \quad \boxed{a=+1} \checkmark$$

$$\xrightarrow{a=1} (2-6)x - 8 < 0 \Rightarrow -4x - 8 < 0 \Rightarrow -(4x+8) < 0 \Rightarrow (x+2) > 0 \Rightarrow x > -2 \quad \boxed{a=1} \quad \boxed{b=-2}$$

$$\xrightarrow{a=-1} (2+6)x - 8 < 0 \Rightarrow 8x - 8 < 0 \Rightarrow 8(x-1) < 0 \Rightarrow x < 1 \quad \boxed{a=-1} \quad \boxed{b=-2}$$

$$\text{حيث } \Rightarrow ax+3 \neq 0 \Rightarrow ax \neq -3 \Rightarrow x \neq -\frac{3}{a}$$

$$\xrightarrow{a=1} x \neq -3 \quad \text{و } \checkmark$$

$$\xrightarrow{a=-1} x \neq 3 \quad \text{و } \checkmark$$

$$\Rightarrow \boxed{a-b = 1 - (-2) = 3}$$

$$|x-r| + |y-1| = r$$

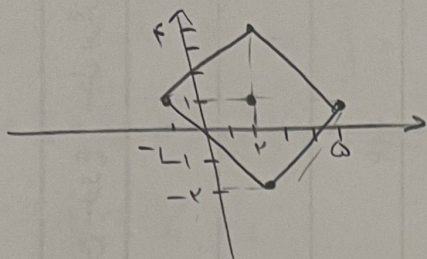
سؤال ٨

$$|y-1| = r - |x-r| \Rightarrow |y-1|^2 = (r - |x-r|)^2 \Rightarrow y-1 = \pm \sqrt{(r - |x-r|)^2}$$

$$\Rightarrow y = \pm \sqrt{(r - |x-r|)^2} + 1 \Rightarrow y = \pm (r - |x-r|) + 1$$

$$\oplus \quad \begin{cases} y-x & x \geq r \\ x+r & x < r \end{cases}$$

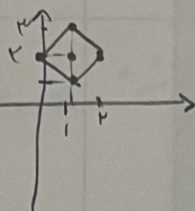
$$\ominus \quad \begin{cases} x-r & x \geq r \\ -x & x < r \end{cases}$$



سؤال ٩

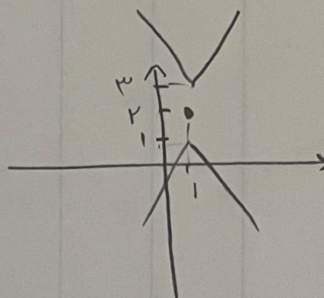
نصف) $|x-1| + |y-1| = 1$

A // K=1 نصف



$$\Rightarrow |x-1| - |y-1| = -r$$

$$x \ominus \Rightarrow |y-1| - |x-1| = r$$



سؤال ١٠

$$y = rx + |ax+b| \quad ax+b=0 \Rightarrow ax=-b \Rightarrow x=-\frac{b}{a}$$

$$-\frac{b}{a}$$

$$rx - ax - b$$

$$(r-a)x - b$$

$$(r-a) > 0$$

$$a < r$$

(I)

$$rx + ax + b$$

$$(r+a)x + b$$

$$a > -r$$

(II)

$$\Rightarrow -r < a < r$$