

۲۰ آفرین!

پارس زیدی

$$f(rx+1) + f(x+r) = rx + f(r)$$

$$y = \sqrt{f(x) - x^2}$$

سوال ۱

$$ax+b \Rightarrow a(rx+1) + b + a(x+r) + b = rx + ra + b$$

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$$rax + a + ax + ra + b = rx + ra + b \Rightarrow rax + ax - rx + b = 0 \Rightarrow$$

$$rax - rx + b = 0 \Rightarrow (ra - r)x + b = 0 \Rightarrow ran + rb = rx + b$$

$$\Rightarrow ran = rx \Rightarrow a = 1 \text{ و } b = 0 \quad y = x$$

$$y = \sqrt{f(x) - x^2} \Rightarrow y = \sqrt{x - x^2} \Rightarrow \begin{cases} x_{\max} = \frac{1}{2} \\ y_{\max} = \frac{1}{2} \end{cases} \quad \sqrt{\left[-\frac{1}{2}, \frac{1}{2}\right]}$$

$$R_f = \left[0, \frac{1}{2}\right] \quad \checkmark$$

سوال ۲ $f(x) = \frac{x^2 - rx}{x+r}$ $R = \{a, b\}$ $f\left(\frac{a+b}{rc}\right) = ?$ $[c, +\infty) - \{1\}$

$$f(x) = \frac{x^2 - rx}{x+r} \Rightarrow D_f = R - \{-r, r\} \quad \begin{cases} a = -r \\ b = r \end{cases} \quad \begin{cases} x+r \neq 0 \Rightarrow x \neq -r \\ \frac{x^2 - rx}{x+r} \neq 1 \end{cases}$$

$$R_f = [c, +\infty) - \{1\}$$

$$y = \frac{x(x^2 - r)}{x+r} = \frac{x(x-r)(x+r)}{x+r} = x^2 - rx \quad \begin{cases} x \leq -\frac{b}{2a} = +1 \\ y_1 = 1 - r = -1 \Rightarrow c = -1 \end{cases}$$

$$f\left(\frac{a+b}{rc}\right) = f(-1) = (-1)^2 - r(-1) = r$$

$$y = \sqrt{x^2 + ax^r + bx - 1} \quad D_f = [a, b] \quad R_f = ?$$

سوال ۳

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$$(x-r)^2 = x^2 - 4x^r + 12x - 1 \Rightarrow \begin{cases} a = -4 \\ b = 12 \end{cases}$$

$$D_f = [-4, 12]$$

$$R_f = [-1, 1]$$

$$y = \sqrt{x^2 + ax^r + bx - 1} = \sqrt{(x-r)^2} = x-r$$

$$y_1 = \left(\frac{x+1}{1-x} \right)^{\frac{1}{5}}$$

$$y_2 = \frac{ax^2-1}{x^2+b}$$

$$\left(\frac{x+1}{1-x} \right) \geq 0 \Rightarrow \frac{-1}{-0+1} = -1 \quad \text{سؤال ٤} \quad \text{Df} = [-1, 1] \quad \text{2}$$

$$R_{y_2} = \begin{cases} x=0 \rightarrow -\frac{1}{b} \\ x \rightarrow \infty \rightarrow a \end{cases} \Rightarrow \begin{cases} \text{أخرج من المنزلة} \rightarrow (-\frac{1}{b}, a) \\ -\frac{1}{b} = -1 \Rightarrow b=1 \end{cases}$$

$$\begin{cases} a=1 \\ b=1 \end{cases}$$

$$a+b=1+1=2$$

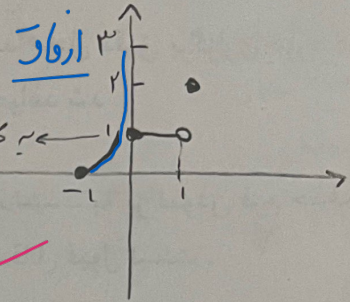
$$\frac{[x]}{x} + \frac{|x|}{x} \text{ دالة } [-1, 1] = ?$$

$$-1 < x < 0 \Rightarrow -\frac{1}{x} + \frac{(-x)}{x} = -\frac{1}{x} - 1 \Rightarrow -\left(\frac{1}{x} + 1\right)$$

$$0 \leq x < 1 \Rightarrow \frac{x}{x} + \frac{x}{x} = 1$$

$$x=1 \Rightarrow \frac{1}{x} + \frac{x}{x} = \frac{1}{x} + 1 = 2$$

$$Df = \mathbb{R} - \{0\} \quad \text{نصف مستقيمة } x \neq 0$$



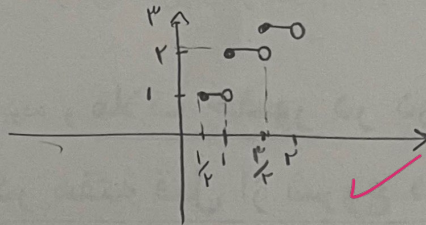
سؤال ٥

$$y = [x] + \left[x + \frac{1}{x} \right] \quad \left(\frac{1}{x}, 2 \right)$$

$$\frac{1}{x} \leq x < 1 \Rightarrow 0 + 1 = 1$$

$$1 \leq x < \frac{x}{x} \Rightarrow 1 + 1 = 2$$

$$\frac{x}{x} \leq x < 2 \Rightarrow 1 + 2 = 3$$



سؤال ٦

$$\left| \frac{x+1}{ax+3} \right| \Rightarrow \left| \frac{x+1}{ax+3} \right| < 1 \Rightarrow (x+1)^2 < (ax+3)^2$$

$$(x+1)^2 - (ax+3)^2 < 0 \Rightarrow (x^2 + 2x + 1) - (a^2x^2 + 6ax + 9) < 0 \Rightarrow$$

$$(1-a^2)x^2 + (2-6a)x - 8 < 0 \Rightarrow 1-a^2 = 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1 \quad \boxed{a=+1} \quad \checkmark$$

$$\xrightarrow{a=1} (1-4)x - 8 < 0 \Rightarrow -3x - 8 < 0 \Rightarrow -(3x+8) < 0 \Rightarrow 3x+8 > 0 \Rightarrow x > -\frac{8}{3} \quad \boxed{a=1} \quad \checkmark$$

$$\xrightarrow{a=-1} (1+4)x - 8 < 0 \Rightarrow 5x - 8 < 0 \Rightarrow 5x < 8 \Rightarrow x < \frac{8}{5} \quad \boxed{a=-1} \quad \checkmark$$

$$\Rightarrow \boxed{a-b = 1 - (-2) = 3}$$

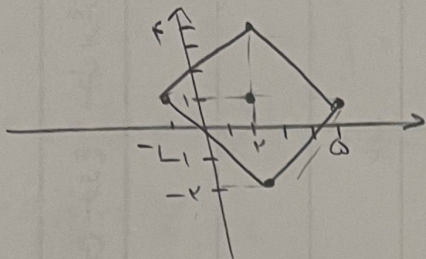
$$|x-r| + |y-1| = r$$

$$|y-1| = r - |x-r| \Rightarrow |y-1|^2 = (r - |x-r|)^2 \Rightarrow y-1 = \pm \sqrt{(r - |x-r|)^2}$$

$$\Rightarrow y = \pm \sqrt{(r - |x-r|)^2} + 1 \Rightarrow y = \pm (r - |x-r|) + 1$$

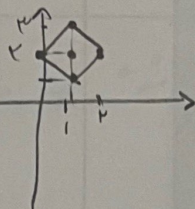
$$\text{وفا } \oplus \begin{cases} y-x & x \geq r \\ x+r & x < r \end{cases}$$

$$\ominus \text{ وفا } \begin{cases} x-r & x \geq r \\ -x & x < r \end{cases}$$



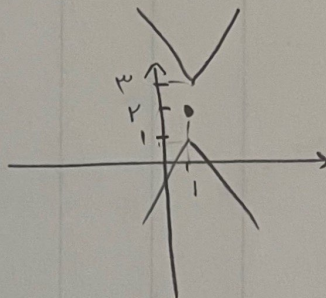
$$\text{نصف } |x-1| + |y-1| = 1$$

A // k=1 نصف



$$\Rightarrow |x-1| - |y-1| = -r$$

$$x \ominus \Rightarrow |y-1| - |x-1| = r$$



$$y = rx + |ax+b| \quad ax+b=0 \Rightarrow ax=-b \Rightarrow x=-\frac{b}{a}$$

$$-\frac{b}{a}$$

$$(r-x-a)x-b$$

$$(r-a)x-b$$

$$(r-a) > 0$$

$$a < r$$

(I)

$$rx+ax+b$$

$$(r+a)x+b$$

$$a > -r$$

(II)

$$\Rightarrow -r < a < r$$