گلرضا

Page : (   )

SUBJECT

Year: Month: Day:

(1)

$f(2x+1)+f(x+2)=3x+f(3) \xrightarrow{x=1}$

$f(3)+f(3)=3+f(3) \Rightarrow \underline{f(3)=3}$ $\qquad f(x)=ax+b$

$f(2x+1)+f(x+2)=3x+3 \Rightarrow a(2x+1)+b+a(x+2)+b=3x+3$

$\Rightarrow 2ax+a+b+ax+2a+b=3x+3 \Rightarrow 3ax+3a+2b=3x+3 \Rightarrow$

$a=1, b=0 \Longrightarrow f(x)=x$

$y=\sqrt{f(x)-x^2} \Rightarrow \sqrt{x-x^2}=y \Rightarrow \frac{-b}{2a}=\frac{-1}{-2}=\frac{1}{2} \rightarrow$

$x-x^2 \xrightarrow{x=1/2} \frac{1}{2}-\frac{1}{4}=\frac{1}{4} \rightarrow \left(-\infty, \frac{1}{4}\right] \xrightarrow{\sqrt{\ }} \left[0, \frac{1}{2}\right]=R_f$

---

(2)

$\frac{x^3-4x}{x+2} \longrightarrow x \neq -2$

$\frac{x^3-4x}{x+2}=\frac{x(x+2)(x-2)}{x+2} \Rightarrow x^2-2x \neq 8 \Rightarrow x^2-2x-8 \neq 0$

$(x-4)(x+2) \neq 0$

$\left.\begin{array}{l} x \neq -2 \\ x \neq 4 \end{array}\right\} \rightarrow \begin{array}{l} a=-2 \\ b=4 \end{array}$

$R_f \longrightarrow x^2-2x \Rightarrow \frac{-b}{2a}=\frac{+2}{2}=1$

$\xrightarrow{x=1} 1-2=-1 \longrightarrow \boxed{c=-1} \longrightarrow R_f=[-1,+\infty)-\{8\}$

$f\left(\frac{a+b}{2c}\right) \Rightarrow f\left(\frac{4-2}{-2}\right)=f(-1)=x^2-2x \Rightarrow 1-(-2)=3$

Year: Month: Day: Page : ( )

(3)

$$y = \sqrt[3]{x^3 + ax^2 + bx - 8} \Longrightarrow \sqrt[3]{(x-2)^3} \Rightarrow \sqrt[3]{x^3 - 6x^2 + 12x - 8}$$

$$a = -6 ,\ b = 12 \longrightarrow D_f = [-6, 12]$$

$$\sqrt[3]{(x-2)^3} \Rightarrow x - 2 = y \;\; \begin{cases} \xrightarrow{x=-6} & -6-2=-8 \\ \xrightarrow{x=12} & 12-2=10 \end{cases} \Rightarrow R_f = [-8, 10]$$

---

(4)

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{5}}$$

$$\longrightarrow D_{y_1} = R_{y_2} \Rightarrow$$ دو تابع وارون یکدیگرند

$$y_2 = \frac{ax^2 - 1}{x^2 + b}$$

$$\frac{x+1}{1-x} \geqslant 0 \longrightarrow \begin{array}{ccccc} & -1 & & 1 & \\ - & & + & & - \end{array} \Rightarrow D_f = [-1, 1) = R_{f_2}$$

$$\left.\begin{array}{l} x^2 = 0 \longrightarrow \frac{-1}{b} = -1 \Rightarrow b = 1 \\ x^2 = +\infty \longrightarrow \frac{a(+\infty)}{(+\infty)} = a = 1 \end{array}\right\} \rightarrow a + b = 2$$

۵

$\frac{[x]}{x}+\frac{|x|}{x} \quad [-۱,۱]$

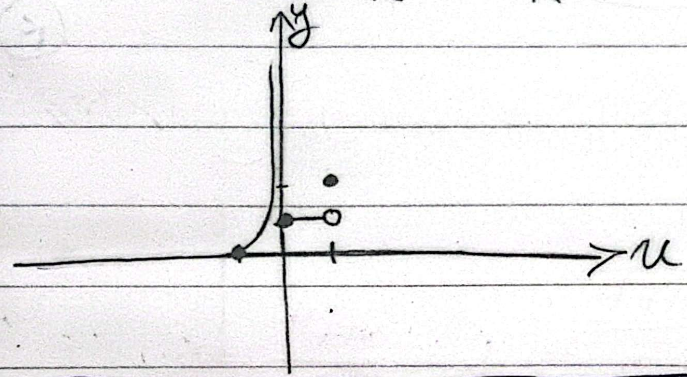
$-۱\leqslant x<۰ \Rightarrow \frac{-۱}{x}+\frac{-x}{x} \Rightarrow \frac{-۱}{x}-۱ \Rightarrow \frac{-۱-x}{x}$

$۰\leqslant x<۱ \Rightarrow \frac{۰}{x}+\frac{x}{x} \Rightarrow ۱$

$x=۱ \Rightarrow \frac{۱}{۱}+\frac{۱}{۱}=۲$

$x=-۱$

$\frac{-۱+۱}{-۱}=۰$



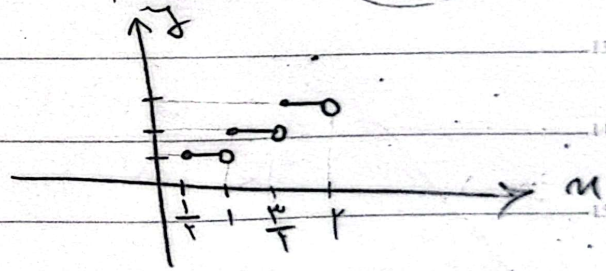
۶

$y=[x]+[x+\frac{۱}{۲}] \quad [\frac{۱}{۲},۲)$

$\frac{۱}{۲}\leqslant x<۱ \Rightarrow y=۱$

$۱\leqslant x<\frac{۳}{۲} \Rightarrow ۱+۱=۲=y$

$\frac{۳}{۲}\leqslant x<۲ \Rightarrow ۱+۲=۳=y$



SUBJECT:
Year: Month: Day:

(7)

$y=\left|\dfrac{x+1}{ax+3}\right| \quad (b,+\infty) \qquad a-b=1+2=3$

$y<1 \Rightarrow \left|\dfrac{x+1}{ax+3}\right|<1 \Rightarrow \dfrac{(x+1)^2}{(ax+3)^2}<1 \Rightarrow$

$(x+1)^2<(ax+3)^2 \Rightarrow x^2+2x+1<a^2x^2+6ax+9$

$(b,+\infty) \longrightarrow a^2=1 \Rightarrow a=1 \Rightarrow x^2+2x+1<x^2+6x+9 \Rightarrow 4x+8>0$

$\checkmark(-2,+\infty)$

$a=-1 \Rightarrow x^2+2x+1<x^2-6x+9 \Rightarrow 8x<8 \qquad b=-2,\ a=1$

$\times(-\infty,1)$

---

(8)

$|x-2|+|y-1|=3 \quad (2,1)$

$-x+2+y-1=3$

$\Rightarrow y=x+2 \quad (2,4) \quad (-1,1)$

$x=2 \Rightarrow 2+2=4$

$y=1 \Rightarrow x+2=1 \Rightarrow x=-1$

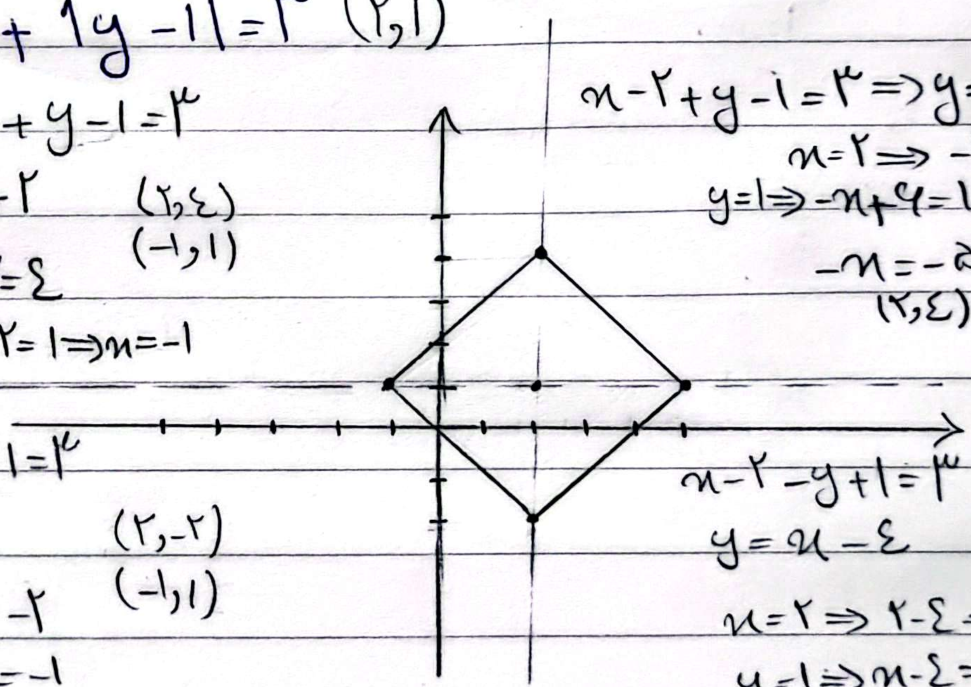
$x-2+y-1=3 \Rightarrow y=-x+6$

$x=2 \Rightarrow -2+6=4$

$y=1 \Rightarrow -x+6=1 \Rightarrow$

$-x=-5 \Rightarrow x=5$

$(2,4),(5,1)$



$-x+2-y+1=3$

$y=-x \quad (2,-2) \quad (-1,1)$

$x=2 \Rightarrow y=-2$

$y=1 \Rightarrow x=-1$

$x-2-y+1=3$

$y=x-4$

$x=2 \Rightarrow 2-4=-2$

$y=1 \Rightarrow x-4=1 \Rightarrow x=5$

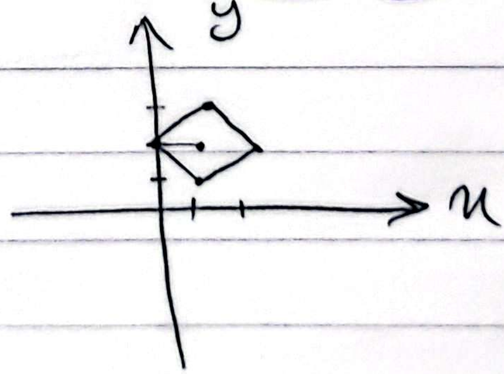
$(5,1),(2,-2)$

---

(9)

الف) $|x-1|+|y-2|=1$

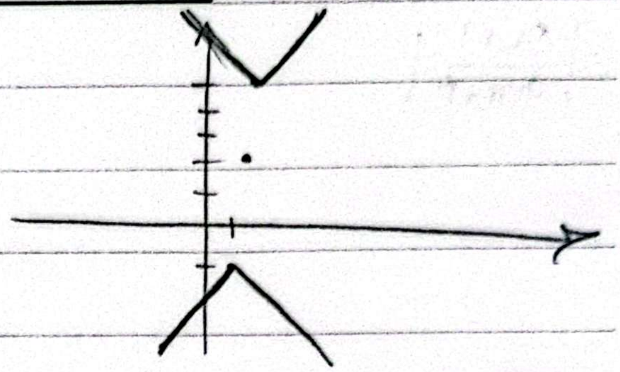
$(1,2)$



ب) $|x-1| - |y-2| = -3$

$(1,2)$

$|y-2| - |x-1| = 3$



$y = 3x + |ax+b| \qquad R_f = \mathbb{R}$ (10)

$$\begin{array}{c|c} & -\frac{b}{a} \\ \hline 3x - ax - b & 3x + ax + b \\ (3-a)x - b & (3+a)x + b \end{array}$$

$-\frac{3b}{a}$

$(3-a)(3+a) > 0$

$9 - a^2 > 0$

$$\begin{array}{ccccc} & -3 & & 3 & \\ \hline - & | & + & | & - \end{array}$$

$|a| < 3$ پس

اگر هم مساوی ۳ یا ۳- قرار دهیم یکی از معادله خط ها عددی ثابت می‌شود.

تابع ثابت

باید یا هر دو شیب مثبت داشته باشند یا منفی که حاصل ضربشان مثبت می‌شود تا بتواند کل R را پوشش دهد.