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$$f(2x+1) + f(x+2) = 3x + f(3) \xrightarrow{f(3)=1} f(3) + f(3) = 3 + f(3) \Rightarrow f(3) = 3 \Rightarrow a(2x+1) + b + a(x+2) + b = 3x + 3 \Rightarrow 2ax + a + b + ax + 2a + b = 3x + 3 \Rightarrow 3ax + 3a + 2b = 3x + 3$$

$$3a = 3 \Rightarrow a = 1, \quad 3a + 2b = 3 \Rightarrow b = 0 \Rightarrow f(x) = x$$

$$y = \sqrt{x - x^2} \quad \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ y_{max} = \frac{1}{2} \end{array} \right. \Rightarrow R = \sqrt{(-\infty, \frac{1}{2}]} \xrightarrow{\text{زیربسیار}} R_f = [0, \frac{1}{2}]$$

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$$f(x) = \frac{x^3 - 2x}{x+2} = \frac{x(x^2 - 2)}{x+2} = x(x-2) = x^2 - 2x \xrightarrow{x=-2} \text{برای یافتن ریشه دیگر} \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0, 2$$

$$x^2 - 2x - 1 = 0 \Rightarrow (x-4)(x+2) = 0 \Rightarrow x = 4 \Rightarrow a, b = -2, 4$$

$$f(x) = x^2 - 2x \quad \left| \begin{array}{l} -\frac{b}{2a} = 1 \\ -1 \end{array} \right. \rightarrow R_f = [-1, +\infty) - \{1\}$$

$$f\left(\frac{4-2}{-2}\right) = f(-1) = -1 + 4 = 3$$

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$$y = \sqrt{x^3 + ax^2 + bx - 1} \xrightarrow{\text{فرض}} y = \sqrt{(x-2)^3} = \sqrt{x^3 - 6x^2 + 12x - 8} \Rightarrow R_y = [-4, 12]$$

$$y = x - 2 = R_f = [-1, 10]$$

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$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} \Rightarrow \frac{x+1}{1-x} \geq 0 \quad \left| \begin{array}{c} -1 \quad 1 \\ x+1 \quad x-1 \end{array} \right| \Rightarrow -1 \leq x < 1$$

$$y_2 = \frac{ax^2 - 1}{x^2 + b} \Rightarrow y_2 x^2 + b y_2 = ax^2 - 1 \Rightarrow ax^2 - y_2 x^2 = b y_2 + 1 \Rightarrow x^2(a - y_2) = b y_2 + 1$$

$$\Rightarrow x^2 = \frac{b y_2 + 1}{a - y_2} \Rightarrow x = \sqrt{\frac{b y_2 + 1}{a - y_2}} \Rightarrow \frac{b y_2 + 1}{a - y_2} \geq 0 \Rightarrow a + b = 3$$

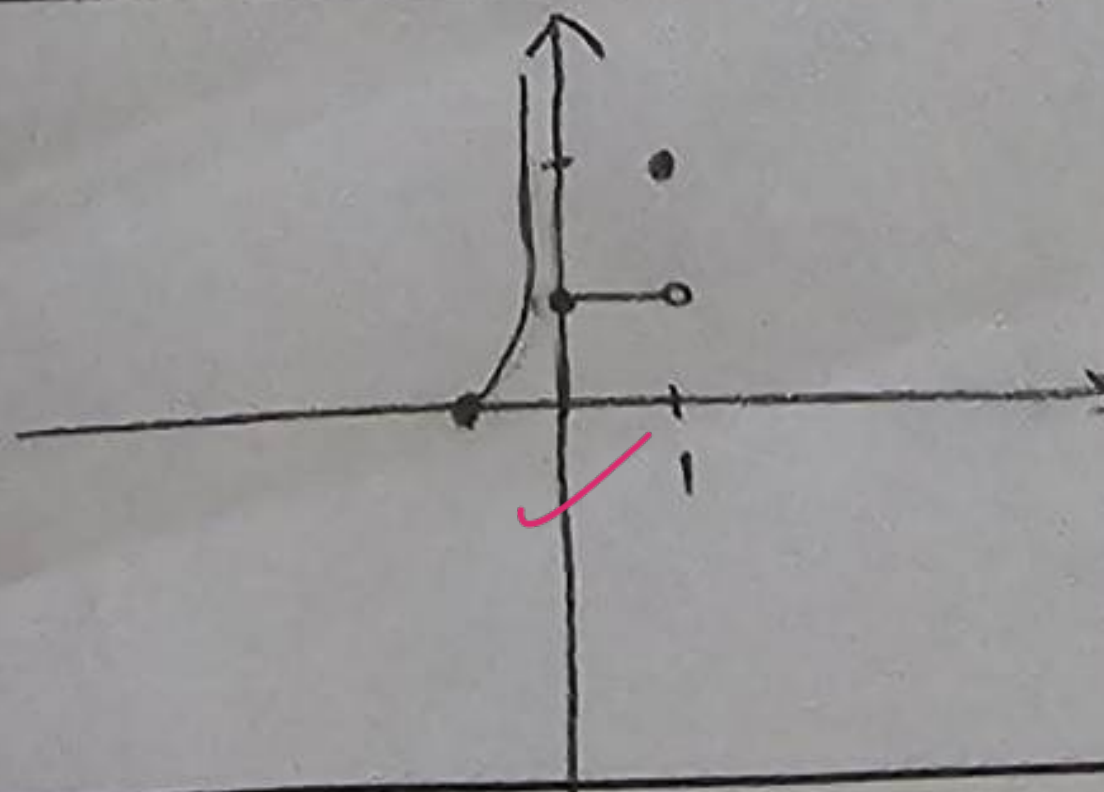
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$$[-1, 1] \quad \frac{[x]}{x} + \frac{|x|}{x}, \quad x \neq 0$$

$$x \in [-1, 0) \Rightarrow y = \frac{-1}{x} + \frac{-x}{x} = -\frac{1}{x} - x$$

$$x \in (0, 1) \Rightarrow y = \frac{0}{x} + \frac{x}{x} = 1$$

$$x = 1 \Rightarrow y = 2$$



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