مشتق ناپذیری

① $f(x)=\sqrt{x(1-|x|)}$

$x\geq 0: \sqrt{x-x^2} \Rightarrow f'=\frac{1-2x}{2\sqrt{x-x^2}}$

$x<0: \sqrt{x+x^2} \Rightarrow f'=\frac{1+2x}{2\sqrt{x+x^2}}$

غیرمشتق‌پذیر: $x: \frac{1}{2}, -\frac{1}{2}, 0$

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② $f'(x)=\frac{1}{2\sqrt{x}}+\frac{-2}{3\sqrt{a-2x}}=\frac{\sqrt{a-2x}-3\sqrt{2x}}{3\sqrt{x}\sqrt{a-2x}}=0$

$\sqrt{a-2x}=2\sqrt{x}$

$a-2x=4x$

$f(0)=\sqrt{a}$

$f(a/2)=\sqrt{a/2}$

$f(a/4)=\sqrt{a/4}+\sqrt{a/2}=\sqrt{\frac{3a}{4}}$

$\sqrt{2\frac{a^2}{12}}=\sqrt{\frac{a^2}{6}}=\sqrt{12}$

$a^2=12\cdot\frac{12a^2}{36}=12$

$x=0, a/3, a/2$

$a=\sqrt{12}=2\sqrt{3}$ - $\boxed{[a]=4}$

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③ $-2<x<2: \frac{x^2(4-x^2)}{x^2-1}=\frac{-x^4+4x^2}{x^2-1} \to f'(x)=(-4x^3+8x)(x^2-1)$

$2<x$

$x<-2: \frac{x^2(x^2-4)}{x^2-1}=\frac{x^4-4x^2}{x^2-1}$

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④ $y=3ax^2+2bx+c \xrightarrow{x=1} 3a+2b+c=0 \Rightarrow 3a+2b=0$

$\xrightarrow{x=0} c=0$

$\xrightarrow{x=1} y=a+b+c=1 \to a+b=1$

$\xrightarrow{x=0} y=d=0$

$2a+2b=2$

$3a+2b=0 \to a=-2,\ b=3$

$3\times-2=-6$

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⑤ $x(x^2-3)=x^3-3x \Rightarrow f'(x)=3x^2-3=0$

$x^2>3$: $x=1$ قابل قبول، $x=-1$ قابل قبول

$\sqrt{3}<x$

$x<-\sqrt{3}$

$x^2<3$: $-\sqrt{3}<x<\sqrt{3}$: $x(3-x^2)=-x^3+3x \Rightarrow f'(x)=-3x^2+3=0$ $x=1$ قابل قبول، $x=-1$ قابل قبول

y'	+	-	+	-	+
y	↗	↘	↗	↘	↗

$x=-1 \to f(-1)=-1(3-1)=\boxed{-2}$

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⑥ $x\geq 0: x^4+2ax^2+b \xrightarrow{y'} 4x^3+4ax=0 \to 4x(x^2+a)$ $\to x=0$, $x=-a$

$x<0: -x^4+2ax^2+b \xrightarrow{y'} -4x^3+4ax=0 \to -4x(x-a)$ $\to x=0$, $x=a \to a=-1$

$1-y=1+3x-\frac{1}{2}\times 1+b$

$1=1-\frac{3}{2}+b \Rightarrow \frac{3}{2}=b$

$\frac{b}{a}=\frac{3/2}{-1/2}=\boxed{-3}$

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⑦ $y'=3x+1=0 \to x=-\frac{1}{3}$

min $\left|\begin{matrix}-1/3\\ 1/3\end{matrix}\right.$

$y_{min}=\frac{3}{9}+\frac{1}{3}-\frac{1}{3}=\frac{3}{9}$

$\frac{1}{9}-\frac{1}{3}=\frac{3}{9}$

$\frac{-a-1+2a-3}{3}=0$

$2a-4=0 \to a=2$

$\frac{2x+2}{3x+1}=0 \to x=-\frac{2}{3}$

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⑧ $y'=\frac{(2bx)(4x^2+ax+1)-(8x+a)(bx^2+v)}{(4x^2+ax+1)^2}$

$\boxed{\frac{b}{a}=\frac{12}{4}=3}$

$\frac{\cancel{8bx^3}+2abx^2+2bx-\cancel{8bx^3}-abx^2-64x-va}{(4x^2+ax+1)^2}$ $\boxed{\begin{matrix}b=12\\ b=3\end{matrix}}$

$=\frac{abx^2+(2b-64)x-va}{(4x^2+ax+1)^2}$

$-\frac{1}{2}$: $\frac{a}{4}-\frac{1}{2}+1=0$ $\boxed{a=4}$

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⑨ $f(x)=\frac{x^4}{x^3-8} \to f'(x)=\frac{4x^3(x^3-8)-3x^2(x^4)}{(x^3-8)^2}=\frac{4x^6-32x^3-3x^6}{(x^3-8)^2}$

$=\frac{x^6-32x^3}{(x^3-8)^2}=\frac{x^3(x^3-32)}{(x^3-8)^2}$

y'	-	0	+	0	-	+
y	↘	↗	↘	↗	

$(-\infty,0]\cup[2,\sqrt[3]{32}]$ بازه‌های کاهش

min مطلق تابع $=2-\sqrt[3]{32}$

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⑩ $f(x)=\frac{x^4-3}{x^2-3} \to f'(x)=\frac{4x^3(x^2-3)-2x(x^4-3)}{(x^2-3)^2}=\frac{4x^5-12x^3-2x^5+6x}{x^4-6x^2+9}$

$\frac{2x(x^4-6x^2+6)}{x^4-6x^2+9}=\frac{2x^5-12x^3+6x}{x^4-6x^2+9}$

y'	-	+	+	0	+	+	+
y	↘	↗	↗		↗	↗	↗

$-\sqrt{3}$ ، 0 ، $\sqrt{3}$ ، $\sqrt{3}$ ، $\sqrt{3}$

$1\sqrt{3}$

مینیمم: $(-\sqrt{3},0)$