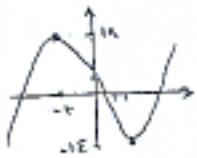
1. $\frac{f(3)-f(1)}{3-1} = \frac{1-\frac{a}{3}-1+\frac{a}{3}}{2} = \frac{a}{3}$ $\quad f'(x) = \frac{a}{x^2} \Rightarrow \frac{a}{3} = \frac{a}{x^2} \Rightarrow x = \pm\sqrt{3}$ $\begin{cases} x = -\sqrt{3} \times \\ x = +\sqrt{3} \checkmark \end{cases}$

2. $2ax^2 - 5x + 18a = x \rightarrow 2ax^2 - 6x + 18a = 0 \xrightarrow{\Delta = 0} 36 - (2a)(18a)(4) = 0 \rightarrow 144a^2 = 36 \Rightarrow a \begin{cases} \frac{1}{6} \times \\ -\frac{1}{6} \checkmark \end{cases}$

$a = \frac{1}{6} \Rightarrow (x-3)^2 = 0 \rightarrow$ ریشه > 0 غ ق ق

3. $y' = 3x^2 - 12 \Rightarrow$ ریشه: $x = \pm 2$

		-2		2	
y'	$+$		$-$		$+$
	$\nearrow$	16	$\searrow$	-16	$\nearrow$



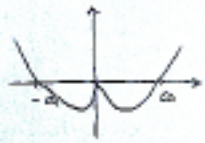
min نسبی $(+2, -16)$

4. ext نسبی $\rightarrow$ تغییر علامت $\Rightarrow$ مشتق $= 0$ $\quad y' = 3x^2 + 2ax - 2b$ $\quad x = 0 \rightarrow -2b = 0 \Rightarrow b = 0$

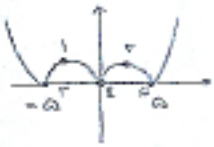
$x = 2 \rightarrow -8 + 4a - 4 = 0 \Rightarrow a = 3$

$y = x^3 + 3x^2 - 4$ $\quad$ ext نسبی $(0, -4)$ $\quad$ ext نسبی $(-2, 0) \Rightarrow$ فاصله: $\sqrt{(0-(-2))^2 + (-4-0)^2} = \sqrt{20}$

5. $f(x) = |x|^2 - 5|x|$



$\Rightarrow y = |f(x)|$

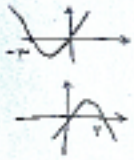


$1, 2 \rightarrow$ max نسبی $\quad m = 2$

$3, 4, 5 \rightarrow$ min نسبی $\quad n = 3$

$\frac{n}{m} = 1.5$

6. $f(x) \begin{cases} x^2 + 3x & x > 0 \\ -x^2 + 3x & x < 0 \end{cases}$



$\Rightarrow$

$\Rightarrow y = |x|$

نقطه بحرانی چون مشتق ندارد

7. $f(x) = \sqrt[3]{x^2}|x-a| \xrightarrow{x<a} -\sqrt[3]{x^2}(x-a) \rightarrow y' = (\frac{2}{3}x^{-\frac{1}{3}})(a-x) + (-1)(x^{\frac{2}{3}}) = 0$

$\frac{2}{3}ax^{-\frac{1}{3}}(a-x) + (-1)(x^{\frac{2}{3}}) = 0 \rightarrow \frac{1}{3}x^{-\frac{1}{3}}(\underbrace{2a - 2x - 3x}_{0}) = 0 \Rightarrow 2a = 5x \rightarrow x = \frac{2a}{5}$

max نسبی $\rightarrow f(\frac{2a}{5}) = \frac{3}{2} \rightarrow -\sqrt[3]{\frac{4a^2}{25}}(\frac{2a}{5} - a) = \frac{3}{2} \rightarrow a\sqrt[3]{\frac{4}{25}a^2} = \frac{3}{2} \xrightarrow{\text{توان } 3} a^3 \cdot \frac{125}{8} = \frac{27}{4} \Rightarrow a = 1.5$

8. $\sqrt{x|x|-x} \begin{cases} \sqrt{x^2-x} & ; x > 0 \\ \sqrt{-x^2-x} & ; x < 0 \end{cases} \Big\} D_f = [-1, 0] \cup [1, +\infty) \Rightarrow f(x) = \begin{cases} \sqrt{x^2-x} & x \geq 1 \\ \sqrt{-x^2-x} & -1 \leq x < 0 \end{cases}$

$\frac{4-0}{8-0} = 1$

$k = 4$
$m = 1$
$n = 0$

$f'(x) = \begin{cases} \frac{2x-1}{2\sqrt{x^2-x}} & x > 1 \\ \frac{-2x-1}{2\sqrt{-x^2-x}} & -1 < x < 0 \end{cases}$ $\quad$ ریشه‌ها: $\frac{1}{2} \times$ ، $1 \times$ $\quad$ ریشه‌ها: $-\frac{1}{2} \times$ ، $-1 \times$

9. نزولی $\rightarrow f'(x) < 0 \Rightarrow \frac{ad-bc}{(cx+d)^2} < 0 \rightarrow ad - bc < 0 \Rightarrow m(m-1) - 2 < 0 \rightarrow m^2 - m - 2 < 0 \Rightarrow -1 \leq m < 2$ $(m \neq 2)$ ①

1 مجانب قائم $\Rightarrow 1 \geq 1 - m \Rightarrow m \geq 0$ ② $\quad \frac{1 \cap 2}{m \in \mathbb{Z}} \rightarrow m = 0, 1$

10. $f(x) \begin{cases} \frac{x}{1-x^2} & x \geq 0 \ (x \neq 1) \\ \frac{x}{1+x^2} & x < 0 \end{cases} \rightarrow f'(x) = \begin{cases} \frac{(1-x^2) - (-2x)(x)}{(1-x^2)^2} & x \geq 0 \\ \frac{(1+x^2) - (2x)(x)}{(1+x^2)^2} & x < 0 \end{cases}$

صورت $= x^2 + 1 \Rightarrow$ ریشه حقیقی ندارد $\Rightarrow$ نقطه بحرانی ندارد

صورت $= 1 - x^2 = 0 \rightarrow x = -1$

نقطه بحرانی $\Rightarrow$ فقط یک نقطه بحرانی است.