تینا جعفری پور — تکلیف ۲۴ — دوازدهم دختر B — ۱۸

۱) $(0,1)\ (3,5) \Rightarrow m=\frac{5-1}{3-0}=\frac{4}{3} \Rightarrow f'(3)=\frac{4}{3}$ ✓ ②

۲) $(-1,1),(2,2) \Rightarrow m=f'(8)=\frac{2-1}{2-(-1)}=\frac{1}{3} \Rightarrow y-2=\frac{1}{3}(x-2) \Rightarrow y=\frac{1}{3}x+\frac{4}{3} \Rightarrow \sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3}$ ریشه مضاعف باید داشته باشد

$\Rightarrow ax-1=\frac{1}{9}x^2+\frac{16}{9}+\frac{8}{9}x \Rightarrow \frac{1}{9}x^2+(\frac{8}{9}-a)x+\frac{25}{9}=0 \Rightarrow \Delta=(\frac{8}{9}-a)^2-4(\frac{1}{9})(\frac{25}{9})=0 \Rightarrow (\frac{8}{9}-a)^2=\frac{100}{81} \Rightarrow \begin{cases}\frac{8}{9}-a=\frac{10}{9} \Rightarrow a=-\frac{2}{9}\\ \frac{8}{9}-a=-\frac{10}{9} \Rightarrow a=2\end{cases}$

$a=-\frac{2}{9} \Rightarrow f'(x)=\frac{a}{2\sqrt{ax-1}}=\frac{-\frac{2}{9}}{2\sqrt{-\frac{2}{9}x-1}} \neq \frac{1}{3}$ مشتق منفی است غ ق ق ②

$a=2 \Rightarrow \frac{2}{2\sqrt{2x-1}}=\frac{1}{3} \Rightarrow x=5 \Rightarrow \boxed{f(5)=3}$

۳) $4y-3x=n \Rightarrow y=\frac{3}{4}x+\frac{n}{4} \Rightarrow \frac{1+m+1}{4}=\frac{3}{4}+\frac{n}{4} \Rightarrow 2+m=3+n \Rightarrow m=n+1$ ②

$y'=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} \Rightarrow \frac{(2+m)(4)-(2+m)}{16}=\frac{3}{4} \Rightarrow 8+4m-2-m=12=3m+6 \Rightarrow m=2 \Rightarrow n=1 \Rightarrow \boxed{m+n=3}$

۴) $3g(x)=\frac{9}{3+\sin x}\qquad f(x)=\frac{(3-\sin x)(9+\sin^2x+3\sin x)}{(3-\sin x)(3+\sin x)}=\frac{\sin^2x+3\sin x+9}{3+\sin x} \Rightarrow h(x)=3g(x)-f(x)=-\frac{(\sin^2x+3\sin x)}{3+\sin x}$

$=-\sin x \Rightarrow h'(x)=-\cos x \Rightarrow h'(\frac{5\pi}{3})=\boxed{-\frac{1}{2}}$ ②

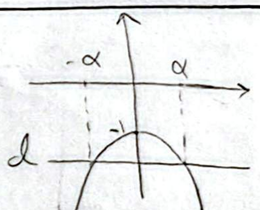
۵) $fog(x)=-\frac{1}{\sqrt[5]{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}} \overset{\sqrt[5]{3}>0}{=} \frac{-1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}}=\frac{-1}{\sqrt[5]{\frac{1}{x^5}}}=-x \Rightarrow (fog(\sqrt[5]{3}))'=\boxed{-1}$ ②

۶) $f'(0)=\lim_{x\to 0}\frac{xg(x)+1-1}{x-0}=\lim_{x\to 0}g(x)$ ②

$f'(x)=2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{2}{(\sin x+1)^2}\right)\times\cos x \Rightarrow f'(0)=2\times(-1)\times 2\times 1=\boxed{-4}$

۷) $y=x^2+1 \underset{\text{قرینه نسبت به محور }x}{\Longrightarrow} y=-x^2-1 \Rightarrow y'=-2x \Rightarrow (-2\alpha)(-2(-\alpha))=-4\alpha^2=-1 \Rightarrow \alpha^2=\frac{1}{4} \Rightarrow \alpha=\pm\frac{1}{2}$

$\Rightarrow |y|=\left|-\frac{1}{4}-1\right|=\left|-\frac{5}{4}\right|=\boxed{\frac{5}{4}}$ ✓ ②



۸) $f(x)=2\sqrt{x}(4x^2+3) \Rightarrow f(b)=2\sqrt{b}(4b^2+3)=ab$

$d: y=ax$

$\Rightarrow \frac{f(b)}{f'(b)}=b=\frac{8\sqrt{b}b^2+6\sqrt{b}}{20\sqrt{b}b+\frac{3\sqrt{b}}{b}}=\frac{8b^2+6}{20b+\frac{3}{b}}$

$f'(x)=\frac{1}{\sqrt{x}}(4x^2+3)+8x(2\sqrt{x}) \Rightarrow f'(b)=\frac{1}{\sqrt{b}}(4b^2+3)+16b\sqrt{b}=a$

$\Rightarrow 8b^2+6=20b^2+3 \Rightarrow 12b^2=3 \Rightarrow b^2=\frac{1}{4} \Rightarrow b=\pm\frac{1}{2}\qquad D_f:[0,+\infty) \Rightarrow f'(\frac{1}{2})=\sqrt{2}(4)+16(\frac{1}{2})(\frac{1}{\sqrt{2}})=\boxed{8\sqrt{2}}=a$ ②

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۱۰ $f \circ g(x) = \left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^3$

۲ $\lim_{x \to (\frac{\sqrt{2}}{2})^-} \frac{\left(\frac{1}{\sqrt{x^2-1}}[1^+]\right)^3 - (2\times 2)^3}{x - \frac{\sqrt{2}}{2}} \overset{\frac{0}{0}}{=} \text{H.O.P} \frac{3\left(\frac{2}{\sqrt{x^2-1}}\right)^2 \times \frac{-2x\times 2}{2\sqrt{x^2-1}} \times \frac{1}{x^2-1}}{1} = \frac{-16\sqrt{2}}{1.1.1} = -16\sqrt{2}$

$y = mn \longrightarrow \dfrac{\sqrt{a}}{-2a^2+a+1} = ma \longrightarrow \dfrac{1}{-2a^2+a+1} = m\sqrt{a}$

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$m\sqrt{a}(-2a^2+a+1) = 1 \longrightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1$ مشتق

$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$

$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \longrightarrow a = -\frac{1}{5}$ ی $a = \frac{1}{2}$ $(a>0)$ ✓

$\varphi_{f(a)} = \dfrac{\sqrt{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \dfrac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$