آزمایی یگا - تکلیف ۲۴:

1. $\left.\begin{matrix} A(3,5) \\ B(0,1) \end{matrix}\right\} m = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$ ۴

2. $\left.\begin{matrix} B(2,2) \\ C(-1,1) \end{matrix}\right\} m = \frac{1}{3} \quad A \left| \begin{matrix} x_0 \\ \sqrt{ax_0 - 1} \end{matrix} \right. \Rightarrow \frac{x}{3} + \frac{4}{3} = \sqrt{ax-1}$

$\Rightarrow y = \frac{x}{3} + \frac{4}{3} \qquad (x+4) = 3\sqrt{ax-1}$

$\Rightarrow x^2 + (8-9a)x + 25 = 0$

$8 - 9a = 10 \Rightarrow -9a = 2 \Rightarrow a = \frac{-2}{9}$

$8 - 9a = -10 \Rightarrow -9a = -18 \Rightarrow a = 2 \rightarrow x^2 - 10x + 25 = 0 \Rightarrow x = 5$ ۳

$f(5) = \sqrt{2(5) - 1} = 3$

3. $y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \xrightarrow{x=1} \frac{4(2+m) - (m+2)}{16} = \frac{3(m+2)}{16}$

$4y - 3x = n \rightarrow m = \frac{-(-3)}{4} = \frac{3}{4} \qquad \frac{3(m+2)}{16} = \frac{3}{4} \Rightarrow m + 2 = 4 \Rightarrow m = 2$

$y = \frac{x^2 + 2x + 1}{x+3} \Rightarrow (1,1) \qquad 4(1) - 3(1) = n \Rightarrow n = 1 \qquad m + n = 2 + 1 = 3$ ۳

4. $g'(x) = \frac{3 + \sin x - 3\cos x}{(3 + \sin x)^2} \qquad f(x) = \frac{(3 - \sin x)(\sin^2 x + 3\sin x + 9)}{(3 - \sin x)(3 + \sin x)} = \frac{\sin^2 x + 3\sin x + 9}{\sin x + 3}$

$(3g(x) - f(x))' = \left(\frac{9 - (9 + 3\sin x + \sin^2 x)}{3 + \sin x}\right)' = (-\sin x)' = -\cos x$

$x = \frac{5\pi}{3} \rightarrow -\cos\frac{5\pi}{3} = \frac{-1}{2}$ ✓ ۲

5. $x > 0 \rightarrow g(x) = \frac{1}{2x^5} \qquad g(x) > 0 \rightarrow f(x) = \frac{-1}{\sqrt[5]{2x}} \Rightarrow fog(x) = \frac{-1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}}$

$\rightarrow fog(x) = -x \Rightarrow (fog)'(x) = -1 \Rightarrow (fog)'(\sqrt[5]{3}) = -1$ ۳

6. $f(x) = \left(\frac{\sin x - 1}{\sin x + 1}\right)^2 = \left(1 + \frac{-2}{\sin x + 1}\right)^2 = 1 + \underbrace{\frac{-4}{\sin x + 1} + \frac{4}{(\sin x + 1)^2}}_{xg(x)}$

$xg(x) = \frac{-4}{\sin x + 1}\left(\frac{\sin x}{\sin x + 1}\right) \Rightarrow$

$\lim_{x \to 0} g(x) = \frac{\frac{-4}{\sin x + 1}\left(\frac{\sin x}{\sin x + 1}\right)}{x} \xrightarrow{\sin x \sim x} \frac{-4}{(\sin x + 1)^2} \Rightarrow \lim_{x \to 0} g(x) = -4$ ۴

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7. قرینه: $-x^2 - 1 \xrightarrow{\text{مشتق}} y' = -2x$ نقاط برخورد با d $A(\alpha, \beta)$ $B(-\alpha, \beta)$

$m_1 = -2(\alpha)$ $m_2 = -2(-\alpha) \Rightarrow m_1 m_2 = -1 \Rightarrow 4\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{2}$ ۴

$\Rightarrow A(\frac{1}{2}, \beta)$ $B(-\frac{1}{2}, \beta)$ فاصله از خط: $\beta \Rightarrow -(\pm\frac{1}{2})^2 - 1 = -\frac{5}{4}$

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8. $f'(x) = \frac{1}{\sqrt{x}}(4x^2 + 3) + (8x)(2\sqrt{x}) = \frac{20x^2 + 3}{\sqrt{x}}$ $(y - y_0) = m(x - x_0)$

$\to (y - (2\sqrt{a})(4a^2 + 3)) = \frac{20a^2 + 3}{\sqrt{a}}(x - a) \xrightarrow{(0,0)} -2\sqrt{a}(4a^2 + 3) = \frac{20a^2 + 3}{\sqrt{a}}(-a)$

$2(4a^2 + 3) = 20a^2 + 3 \Rightarrow a^2 = \frac{1}{4}$ $m = \frac{20a^2 + 3}{\sqrt{a}} = \frac{20(\frac{1}{4}) + 3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$ ۴

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10. $(f \circ g(x))' = g'(x) f'(g(x))$ $g'(x) = \frac{-1}{2}(x^2 - 1)^{-\frac{3}{2}} \times 2x$

$g'(\frac{\sqrt{5}}{2}) = \frac{1}{\sqrt{(\frac{5}{4})^3 - 1}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$

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$f'(x) = f'(2) = ((2x^3))' = 24x^2 = 24(2^2) = 24 \times 4$

$(f \circ g(x))' = -4\sqrt{5} \times 24 \times 4 \xrightarrow{\div -48\sqrt{5}} 8$ برابر

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$$y = mx \rightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \rightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \rightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1$$ مشتق

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \rightarrow a = -\frac{1}{5}$$ یا $$a = \frac{1}{2} \checkmark \quad (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$